Tracer Budgets in ECCO—Part I: Overview and Some Applications

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What Drives Changes in Ocean Heat & Freshwater Content?
Sea-Surface Temperature & the "Slowdown"

Fyfe et al. (2016), Nat. Clim. Change
Decadal Variability in the Subpolar North Atlantic

North East Atlantic mean (50°–10° W; 35°–65° N)

T700 (°C) and S700 (PSU×10) and DLS density (kg m⁻²)

Year

Robson et al. (2016), Nat. Geosci.
Ongoing Changes in Global-Mean Steric Sea Level

Roemmich and Gilson (2009), Prog. Oceanogr.
Figure provided by https://sealevel.nasa.gov/
Conservation Laws (in a rescaled coordinate $z^*$)

\[
\frac{\partial (s^* \theta)}{\partial t} = -\nabla_{z^*} (s^* \theta \mathbf{v}_{\text{res}}) - \frac{\partial (\theta w_{\text{res}})}{\partial z^*} + s^* F_{\theta} + s^* D_{\theta}
\]

\[
\frac{\partial (s^* S)}{\partial t} = -\nabla_{z^*} (s^* S \mathbf{v}_{\text{res}}) - \frac{\partial (S w_{\text{res}})}{\partial z^*} + s^* F_{S} + s^* D_{S}
\]

\[
z^* \doteq \frac{z - \eta(x, y, t)}{H(x, y) + \eta(x, y, t)} H(x, y) , \quad s^* \doteq 1 + \eta/H
\]
Pop Quiz

Which of the following processes can, in principle, effect changes in **globally averaged** steric sea level?

a. Sea-surface heat exchanges
b. Internal ocean heat advection
c. Small-scale mixing of heat
Difficulties of Closing Budgets With Ocean Data & Re-analyses
Observational Challenges

- Comprehensive diagnosis of budgets requires data that are uncertain and hard (if not impossible) to make.

- Example of the 2009-2010 Subtropical North Atlantic “Cold Anomaly” (Bryden et al. 2014; Cunningham et al. 2013; Roberts et al. 201)

Some data assimilation methods constrain to data by imparting non-physical sources & sinks (e.g., of heat, momentum, etc.) to model solutions, precluding any meaningful budget analysis.
Diagnosing Budgets
Using ECCO Version 4
Discretizing the Primitive Equations

\[
\frac{\partial (s^* \theta)}{\partial t} = -\nabla_z^* (s^* \mathbf{v}_{res}) - \frac{\partial (\theta w_{res})}{\partial z^*} + s^* F_\theta + s^* D_\theta
\]

\[
\frac{s^{n+1} \theta^{n+3/2} - s^n \theta^{n+1/2}}{\Delta t} = A \left( \theta, \mathbf{u}^{n+1} + \mathbf{u}_b^{n+1} \right) + s^n \left( F_{\theta}^{n+1} + D_{\sigma,\theta}^{n+1/2} + D_{\perp,\theta}^{n+3/2} \right)
\]
### Required MITgcm Diagnostic Output

<table>
<thead>
<tr>
<th>Diagnostic</th>
<th>Time</th>
<th>Description (Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETAN</td>
<td>Snapshot</td>
<td>Surface height anomaly (m)</td>
</tr>
<tr>
<td>THETA</td>
<td>Snapshot</td>
<td>Potential temperature (°C)</td>
</tr>
<tr>
<td>TFLUX</td>
<td>Average</td>
<td>Total heat flux (W m(^{-2}))</td>
</tr>
<tr>
<td>oceQsw</td>
<td>Average</td>
<td>Net shortwave radiation (W m(^{-2}))</td>
</tr>
<tr>
<td>ADVr_TH</td>
<td>Average</td>
<td>Vertical advective flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>ADVx_TH</td>
<td>Average</td>
<td>Zonal advective flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>ADVy_TH</td>
<td>Average</td>
<td>Meridional advective flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>DFrI_TH</td>
<td>Average</td>
<td>Implicit vertical diffusive flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>DFrE_TH</td>
<td>Average</td>
<td>Explicit vertical diffusive flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>DFxE_TH</td>
<td>Average</td>
<td>Explicit zonal diffusive flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
<tr>
<td>DFyE_TH</td>
<td>Average</td>
<td>Explicit meridional diffusive flux of pot. temp. (°C m(^{3}) s(^{-1}))</td>
</tr>
</tbody>
</table>

MITgcm diagnostics required to evaluate the grid-cell heat budget
Algorithm

1: for $t = t_1, t_2, \ldots, t_{T-1}, t_T$ do

2: \quad $U_{i,j,k} = \text{ADVx_TH} \{t\}$ \quad $\triangleright$ Loop over $T$ time steps (months) $t$
3: \quad $V_{i,j,k} = \text{ADVy_TH} \{t\}$ \quad $\triangleright$ 3-D average zonal advection over month $t$
4: \quad $W_{i,j,k} = \text{ADVz_TH} \{t\}$ \quad $\triangleright$ 3-D average meridional advection over month $t$
5: \quad $\mathcal{U}_{i,j,k} = \text{DFxE_TH} \{t\}$ \quad $\triangleright$ 3-D average vertical advection over month $t$
6: \quad $\mathcal{V}_{i,j,k} = \text{DFy_E_TH} \{t\}$ \quad $\triangleright$ 3-D average zonal diffusion over month $t$
7: \quad $\mathcal{W}^E_{i,j,k} = \text{DFy_E_TH} \{t\}$ \quad $\triangleright$ 3-D average meridional diffusion over month $t$
8: \quad $\mathcal{W}^I_{i,j,k} = \text{DFy_I_TH} \{t\}$ \quad $\triangleright$ 3-D average vertical diffusion (explicit) over month $t$
9: \quad $N^{(0)}_{i,j} = \text{ETAN} \{t - \Delta t\}$ \quad $\triangleright$ 3-D average vertical diffusion (implicit) over month $t$
10: \quad $N^{(f)}_{i,j} = \text{ETAN} \{t\}$ \quad $\triangleright$ 2-D surface height snapshot at start of month $t$
11: \quad $T^{(0)}_{i,j,k} = \text{THETA} \{t - \Delta t\}$ \quad $\triangleright$ 2-D surface height snapshot at end of month $t$
12: \quad $T^{(f)}_{i,j,k} = \text{THETA} \{t\}$ \quad $\triangleright$ 3-D temperature snapshot at start of month $t$
13: \quad $v_{i,j,k} = h_{i,j,k} A_{i,j} \Delta z_k$ \quad $\triangleright$ 3-D temperature snapshot at end of month $t$
14: \quad $v_{i,j,k} = h_{i,j,k} A_{i,j} \Delta z_k$ \quad $\triangleright$ Grid volume
Algorithm

14: for \( i = i_1, i_2, \ldots, i_{I-1}, i_I \) do  
   ▷ Loop over \( I \) longitude cells \( i \)
15:     for \( j = j_1, j_2, \ldots, j_{J-1}, j_J \) do  
       ▷ Loop over \( J \) latitude cells \( j \)
16:         \( s_{i,j}^{(0)} = \left( 1 + N_{i,j}^{(0)} / H_{i,j} \right) \)
17:         \( s_{i,j}^{(f)} = \left( 1 + N_{i,j}^{(f)} / H_{i,j} \right) \)
18:     for \( k = k_1, k_2, \ldots, k_{K-1}, k_K \) do  
       ▷ Loop over \( K \) vertical cells \( k \)
19:         \( G^{\theta, \text{tot}}_{i,j,k} = \left( T_{i,j,k}^{(f)} s_{i,j}^{(f)} - T_{i,j,k}^{(0)} s_{i,j}^{(0)} \right) / \Delta t \)
20:         \( G^{\theta, \text{advH}}_{i,j,k} = (U_{i,j,k} - U_{i+1,j,k} + V_{i,j,k} - V_{i,j+1,k}) / v_{i,j,k} \)
21:         \( G^{\theta, \text{diffH}}_{i,j,k} = (U_{i,j,k} - U_{i+1,j,k} + V_{i,j,k} - V_{i,j+1,k}) / v_{i,j,k} \)
22:         \( G^{\theta, \text{advV}}_{i,j,k} = [(1 - \delta_{i,k,K}) W_{i,j,k+1} - W_{i,j,k}] / v_{i,j,k} \)
23:         \( G^{\theta, \text{diffV}}_{i,j,k} = [(1 - \delta_{i,k,K}) (W_{i,j,k+1}^{E} + W_{i,j,k+1}^{I}) - W_{i,j,k}^{E} - W_{i,j,k}^{I}] / v_{i,j,k} \)
24:         \( G^{\theta, \text{adv}}_{i,j,k} = G^{\theta, \text{advH}}_{i,j,k} + G^{\theta, \text{advV}}_{i,j,k} \)
25:         \( G^{\theta, \text{diff}}_{i,j,k} = G^{\theta, \text{diffH}}_{i,j,k} + G^{\theta, \text{diffV}}_{i,j,k} \)
26:     end for
27: end for
28: end for
Example of Closure

Heat budget at Lon=-98.5, Lat=-0.19937

Figure 2: Heat budget for an arbitrary surface grid cell. Top panel shows the individual terms in the budget equation. Bottom panel shows the LHS, RHS, and difference between LHS and RHS terms in the budget. The good agreement between RHS and LHS (e.g., the ratio of the standard deviation of the residual to the standard deviation of the tendency here is $O(5 \times 10^{-6})$) demonstrates practical closure of the budget.

To demonstrate the relevance of this term in the global ocean heat budget, the horizontally averaged value of the geothermal heating is $0.095 \, \text{W m}^{-2}$. This is not negligible relative to the average heating of the ocean in the ECCOv4 Release 3 solution over 1992–2015 ($0.237 \, \text{W m}^{-2}$). To incorporate the geothermal contribution into the heat budget, one simply considers the ocean bottom grid cells, and normalizes the heat flux by reference density, specific heat capacity, and the vertical thickness of the bottom grid cell, as sketched in Algorithm.

4.3 Salt Conservation

The salt conservation equation in $z$ is (see equation 5 in Forget et al. 2015),

$$
\frac{\partial}{\partial t} (S_S) |_{\{z\}}^{G_{S,tot}} = r_{z} (s_S v_{res}) - \frac{\partial}{\partial z} (S_w v_{res}) |_{\{z\}}^{G_{S,adv}} + s_F |_{\{z\}}^{G_{S,forc}} + s_D |_{\{z\}}^{G_{S,diff}}, \quad (8)
$$
For more details, come to tomorrow’s tutorial and “hands-on” lab session—
- Friday, May 24th
- 3:00-4:00 PM; 4:30-6:30 PM

A full guide is available and downloadable—
- https://dspace.mit.edu/handle/1721.1/111094
A Few Illustrative Vignettes
Vignette #1—
Sea Level & Heat Content in the Tropical Indian Ocean
Sea Level & Heat Content in the Tropical Indian

SSH trends 1993–2015

SSH trends 1993–2003

SSH trends 2004–2014

Thompson et al. (2016), J. Geophys. Res. Oceans
Sea Level & Heat Content in the Tropical Indian

Thompson et al. (2016), J. Geophys. Res. Oceans
Sea Level & Heat Content in the Tropical Indian Ocean

Spatial means over Indian Ocean north of 5°S

- Aviso SSH
- ECCOv4 SSH
- ECCOv4 $\langle \theta \rangle_{\text{ENI}}$

Thompson et al. (2016), J. Geophys. Res. Oceans
Sea Level & Heat Content in the Tropical Indian

A. Time Series of Major Budget Terms

B. Std. Dev. of Advection Terms

Thompson et al. (2016), J. Geophys. Res. Oceans
Sea Level & Heat Content in the Tropical Indian

Thompson et al. (2016) J. Geophys. Res. Oceans

\[ V_{\text{cec}}(t) = \frac{1}{\beta \rho_0} \int_{x_n}^{x_n} \frac{\tau_x(x, 3^\circ S, t) - \tau_x(x, 3^\circ N, t)}{L_y} \, dx, \]
Sea Level & Heat Content in the Tropical Indian

Thompson et al. (2016) J. Geophys. Res. Oceans
Vignette #2—Decadal Variability in the Subpolar North Atlantic Ocean
Regional Heat Content in the North Atlantic

Piecuch et al. (2017), J. Geophys. Res. Oceans
Regional Heat Content in the North Atlantic

A. Observed & estimated heat content changes

B. Ocean heat content budget

Piecuch et al. (2017), J. Geophys. Res. Oceans
A. OHC variance due to wind stress

B. OHC variance due to buoyancy flux

Foukal and Piecuch, in prep.
Forcing of Eastern Subpolar Gyre Budget

Regional Heat Content in the North Atlantic

Foukal and Piecuch, in prep.
Forcing of Subpolar Gyre/Labrador Sea Budget

Regional Heat Content in the North Atlantic

Foukal and Piecuch, in prep.
Vignette #3—
Global-Mean Steric
Sea Level Change
Global-Mean Steric Sea Level

**A. Global-mean ocean temperature changes**

**B. Global-mean ocean temperature budget**
Global-Mean Steric Sea Level

A. Global-mean steric height changes

B. Global-mean steric height budget
Global-Mean Steric Sea Level

Nonlinear Equation of State Effects

A. Thermal Expansion Coefficient ($10^{-4}$/deg C)

B. Surface Heat Flux (Sea Level Eq.) ($10^8$ m/s)

Griffies and Greatbatch (2012), Ocean Modell.
Global-Mean Steric Sea Level

Nonlinear Equation of State Effects
Other examples (that sadly I don’t have time to discuss)

- Heat budgets—Kim et al. (2004, 2007); Lee et al. (2004); Nie et al. (2013); Buckley et al. (2014); Tamsitt et al. (2016); Ponte & Piecuch (2018); Su et al. (2018); Asbjørnsen et al. (2019) ...

- Salt & salinity budgets—Qu et al. (2011, 2013); Vinogradova & Ponte (2013, 2017); Gao et al. (2014) Ponte & Vinogradova (2017) ...

- Regional steric height budgets—Piecuch & Ponte (2011, 2012, 2013) ...

- Momentum budgets—Sonnewald et al. (2019) ...

- ...
Oh, Right ... Pop Quiz

Which of the following processes can, in principle, effect changes in **globally averaged** steric sea level?

a. Sea-surface heat exchanges
b. Internal ocean heat advection
c. Small-scale mixing of heat
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c. Small-scale mixing of heat
Summary

- Budget analysis is a powerful tool for understanding and identifying the mechanisms of oceanic change.

- Difficulties with observations often preclude closed budget analyses.

- The data-constrained and physically consistent nature of the ECCO state estimates allows for meaningful budget attribution of observed oceanic changes.
Thank you.