Adjoint-Derived Drivers of Ocean Variability

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How does ocean (temperature) variability arise?

- External forcing to the climate system
- Advection and mixing of anomalies
- Integration of stochastic atmospheric variability
- Physically coupled modes (ENSO)

What is the role of the atmosphere in driving ocean variability in a region?

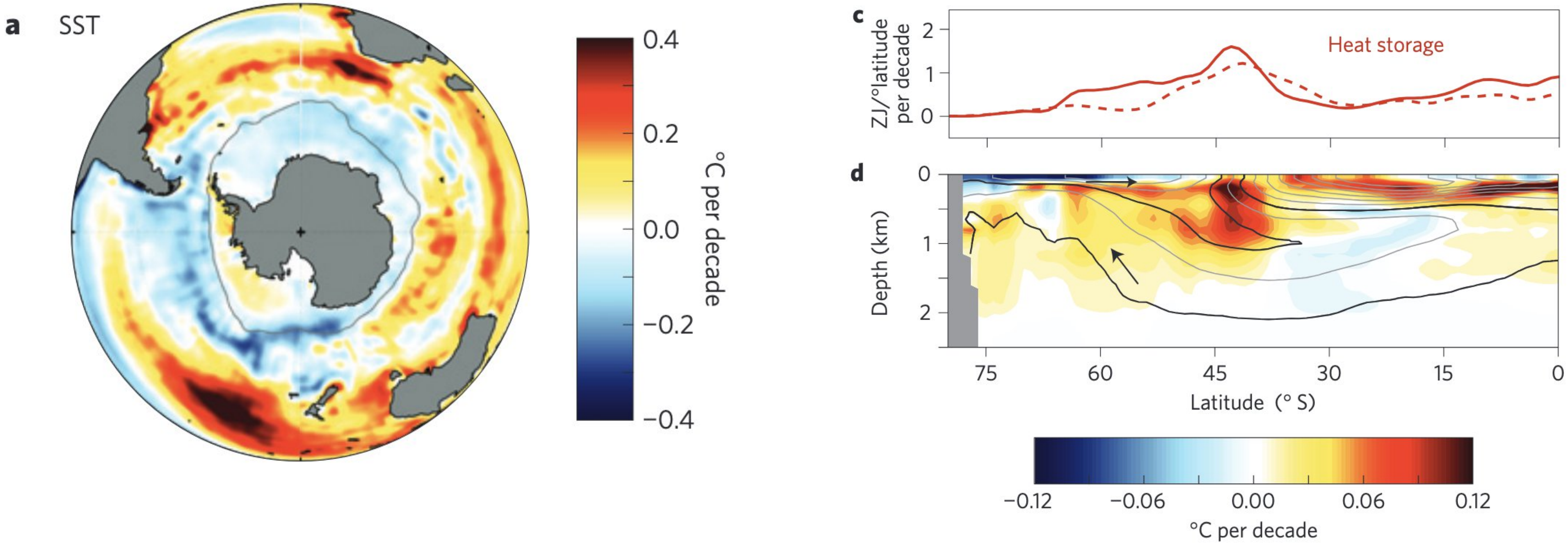
Quantity of Interest (QoI): SST in the Southern Ocean

- Multidecadal cooling trend (1979-2010s) not seen in CMIP models (but it is in ECCO)
- Strong influence of the Southern Annual Mode
- Observed and forecasted strengthening and poleward shift of circumpolar westerlies

What is the role of the atmosphere in driving ocean variability in a region?

Quantity of Interest (QoI): SST in the Southern Ocean

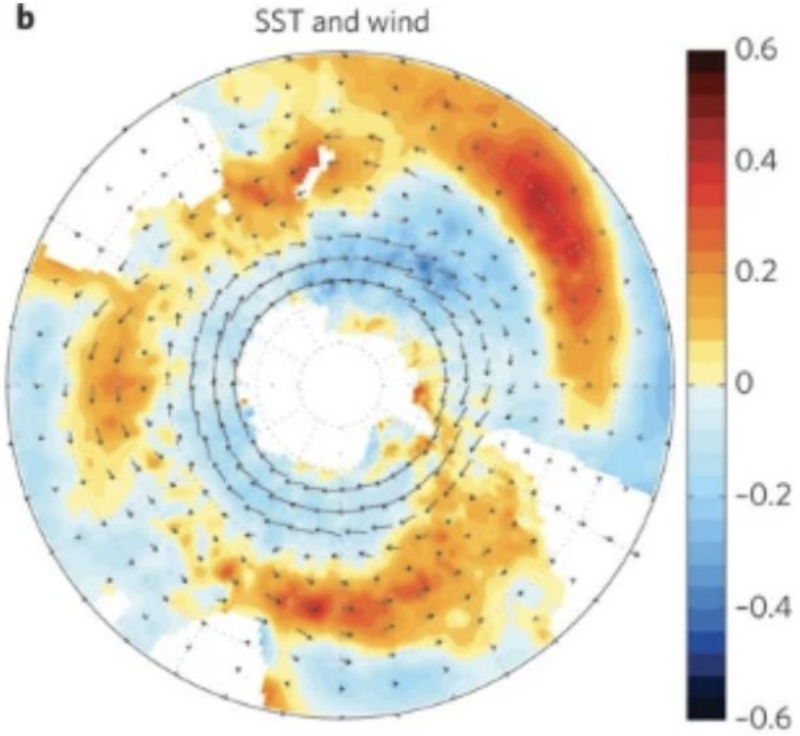
- Armour et al. (2016): Mean advection plays leading role



What is the role of the atmosphere in driving ocean variability in a region?

Quantity of Interest (QoI): SST in the Southern Ocean

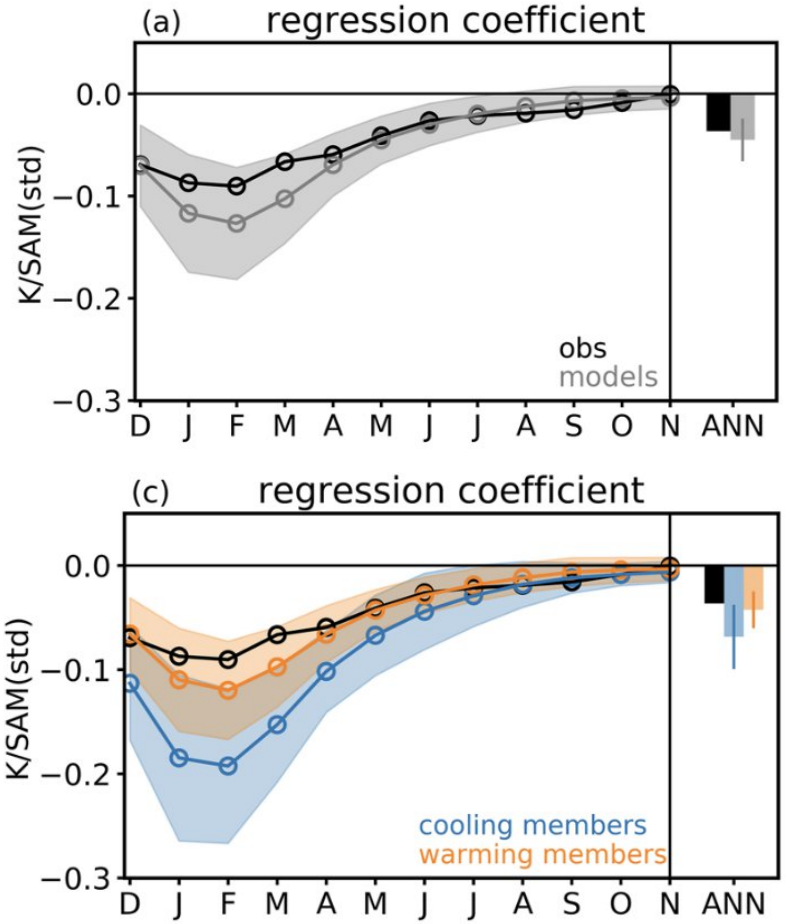
- Thomspon, D.W.J et al. (2011): Regression of the summertime SAM onto SST and wind shows cooling associated with strengthened westerlies
- Positive SAM associated with ozone depletion proposed as driver of cooling



What is the role of the atmosphere in driving ocean variability in a region?

Quantity of Interest (QoI): SST in the Southern Ocean ($\overline{SST}$)

- Dong et al. (2023): Models agree with observations in seasonal SAM-SST relationship, but this cannot explain Southern Ocean cooling



Approach: using the MITgcm adjoint (through EMU), can we recover a leading-order atmospheric forcing that explains $\overline{SST}$?

The adjoint computes derivatives of a quantity of interest (e.g. Southern Ocean SST) to controls (fluxes, ICs, mixing coefficients)

$$\mathbf{s} = \frac{\partial J}{\partial \phi}$$

"Quantity of interest"
Any function of the model state (e.g., AMOC strength at 26N)

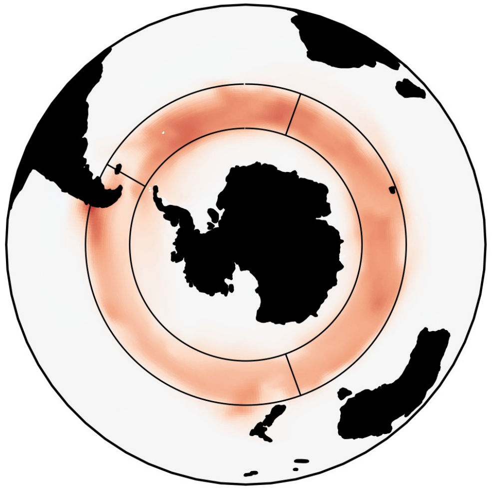
"Controls"
Vector of ocean model inputs

Adjoint sensitivity
How much will changing ϕ change J?

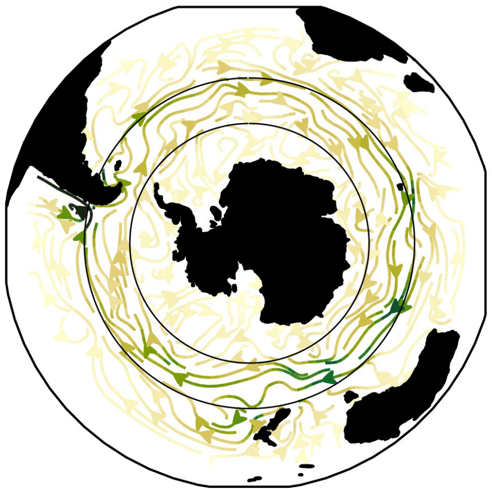
Courtesy Dan Amrhein

Approach: using the MITgcm adjoint, can we recover a leading-order atmospheric forcing that explains $\overline{SST}$?

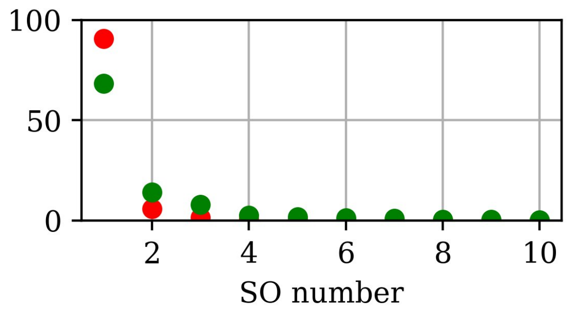
Using these gradients, we can compute the leading *Stochastic Optimals* with respect to the ocean QoI ("**what the ocean wants**") (Kleeman and Moore, 1996)



Leading SO of heat flux



Leading SO of wind stress

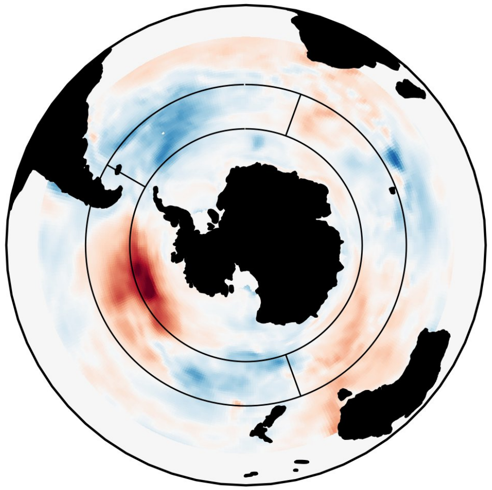


% variance in *linear sensitivity* explained (heat flux, wind stress)

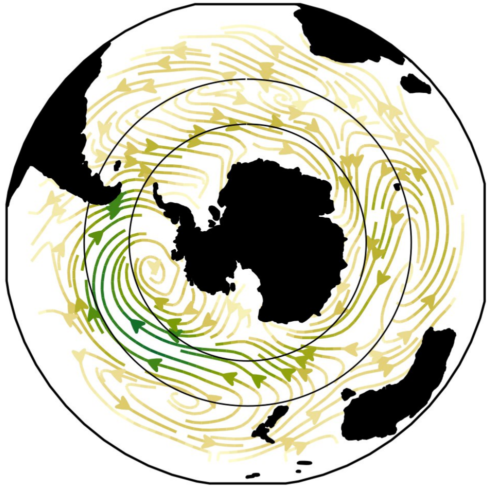
Rosenberg et al., *under review*

Approach: using the MITgcm adjoint, can we recover a leading-order atmospheric forcing that explains $\overline{SST}$?

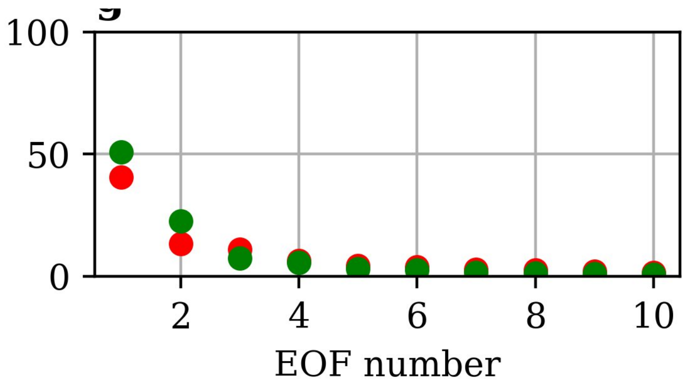
These patterns associated with lag-series—directly analogous to EOFs and associated PC time series ("**what the ocean gets**")



Leading EOF of heat flux from ECCO forcing



Leading EOF of wind stress



% variance in *air-sea flux* explained (heat flux, wind stress)

Rosenberg et al., *under review*

The EOF and SO decompositions can tell us to first order what is happening in the atmosphere and what, in theory, would optimally perturb the ocean state. But neither can point to what in the atmosphere is driving the ocean.

Dynamics-weighted Principal Component Analysis: Adjoint-informed patterns that optimally drive ocean variance

$$\mathbf{S} = \frac{\partial J}{\partial \boldsymbol{\phi}}$$

Sensitivity matrix: dimensions (space X lag)

Courtesy Dan Amrhein

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$$\delta J(t) \approx \sum_{i=1}^{N_\tau} \mathbf{s}(\tau_i)^T \delta\boldsymbol{\phi}(t - \tau_i)$$

Linear change in $\overline{SST}$

Courtesy Dan Amrhein

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Linear change in $\overline{SST}$

$$\sigma_J^2 = \left\langle (\delta J(t))^2 \right\rangle$$

Variance in $\overline{SST}$

Courtesy Dan Amrhein

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$$\sigma_J^2 = \left\langle (\delta J(t))^2 \right\rangle$$

$$\approx \sum_{i=1}^{N_\tau} \sum_{j=1}^{N_\tau} \mathbf{s}(\tau_i)^T \left\langle \delta\phi(t-\tau_i)\,\delta\phi^T(t-\tau_i) \right\rangle \mathbf{s}(\tau_j)$$

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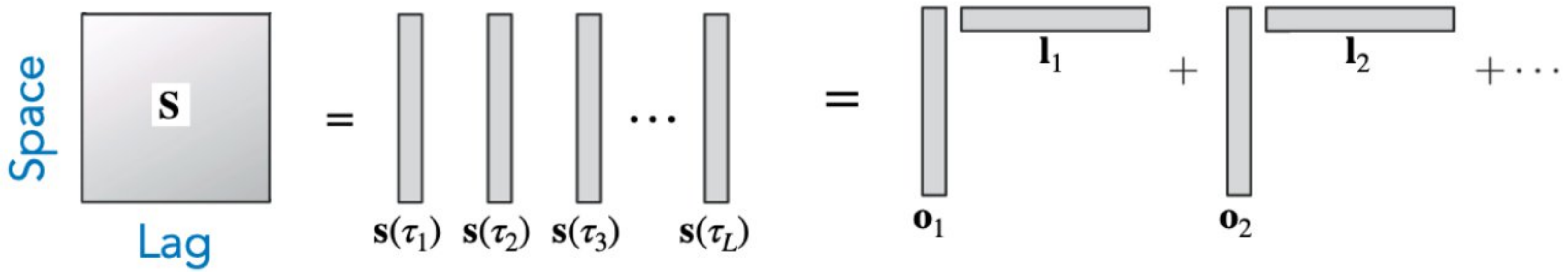
Variance in $\overline{SST}$

$$\approx \mathrm{tr}\left(\left(\mathbf{S}^{\mathrm{T}} \boldsymbol{\phi} \right)^2 \right)$$

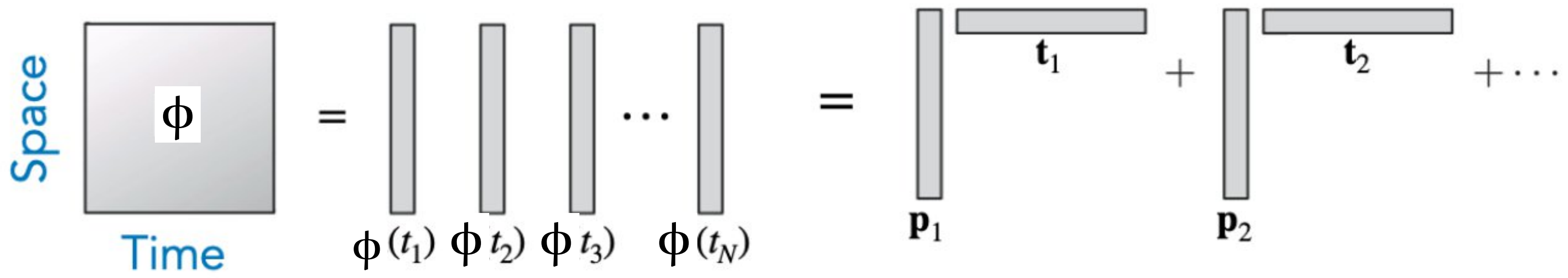
Approximation with stationary atmospheric white noise

Courtesy Dan Amrhein

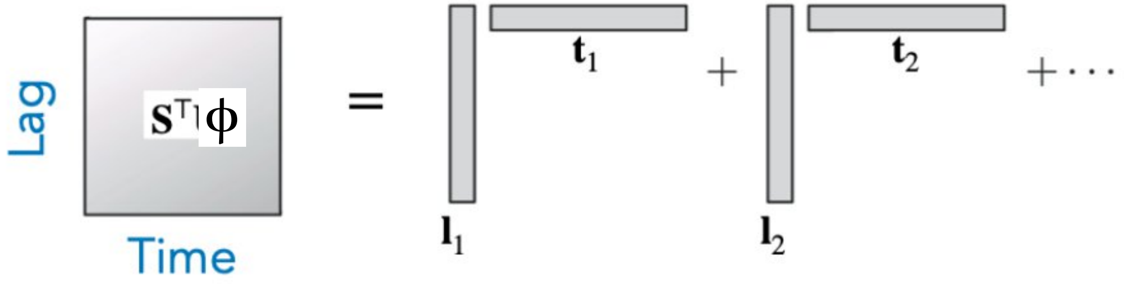
The leading **stochastic optimal** most efficiently excites ocean variance.



The leading **EOF** dominates atmospheric variance.

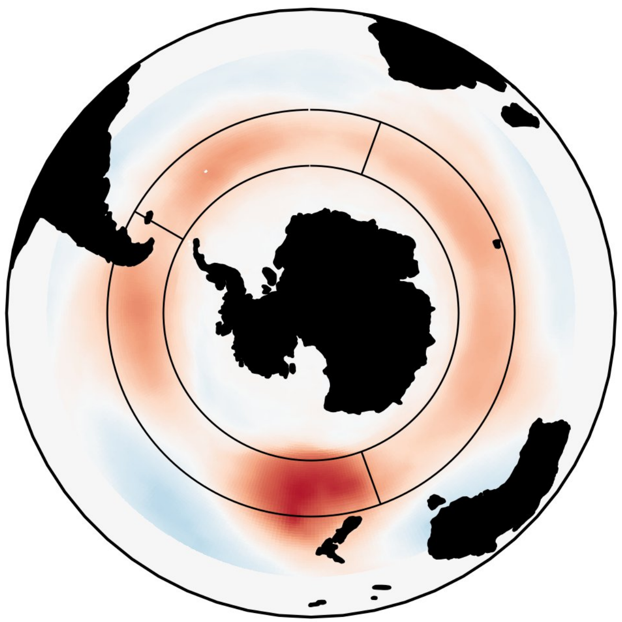


The leading **"EDF"** dominates **ocean** variance.
Patterns are not correlated in time, but may be in space.
(AKA "balanced truncation")

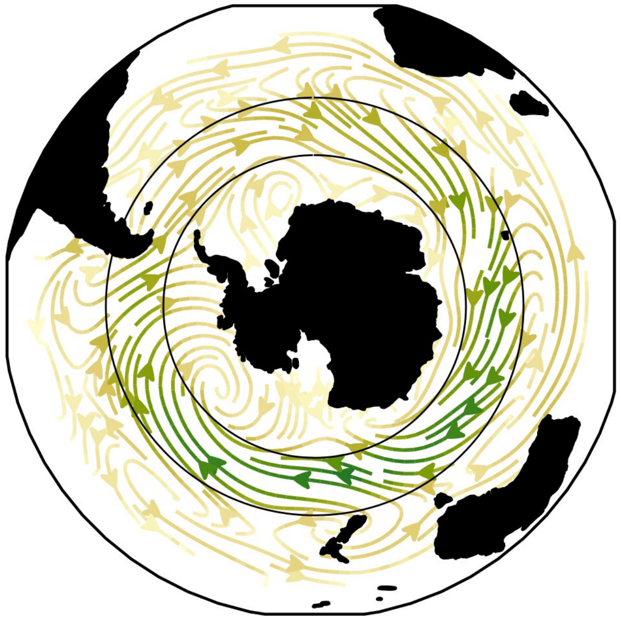


Courtesy Dan Amrhein

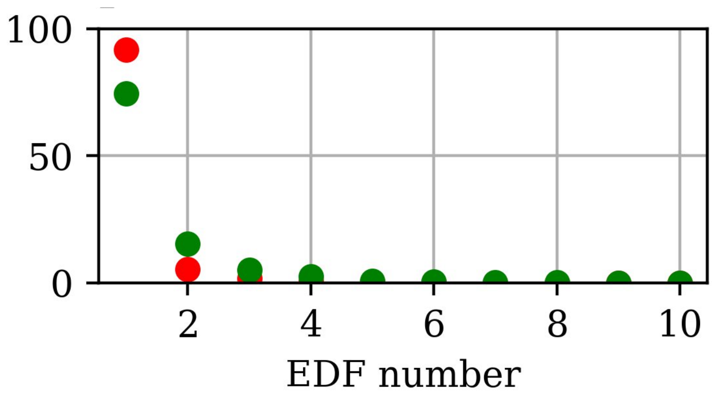
What do the EDFs look like? Largely, we recover the SAM regression onto winds.



Leading EDF of heat flux



Leading EDF of wind stress



% variance in *linear reconstruction* explained
(heat flux, wind stress)

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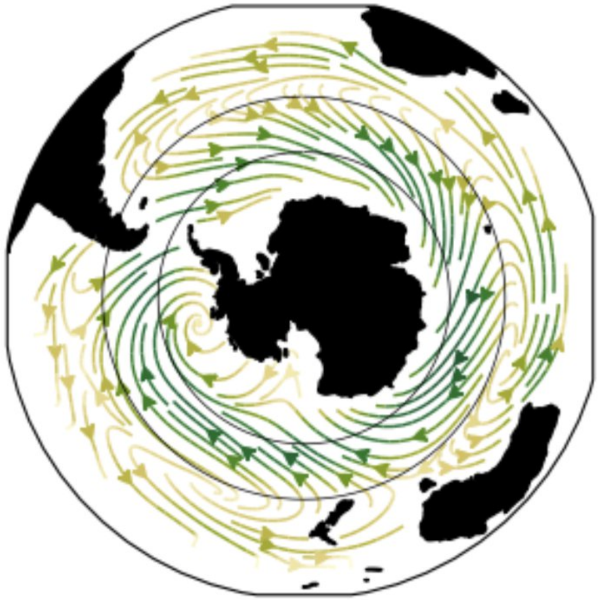
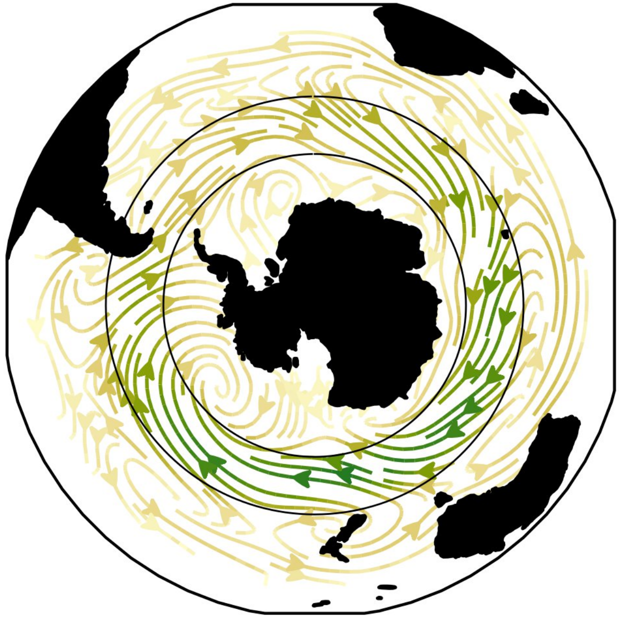
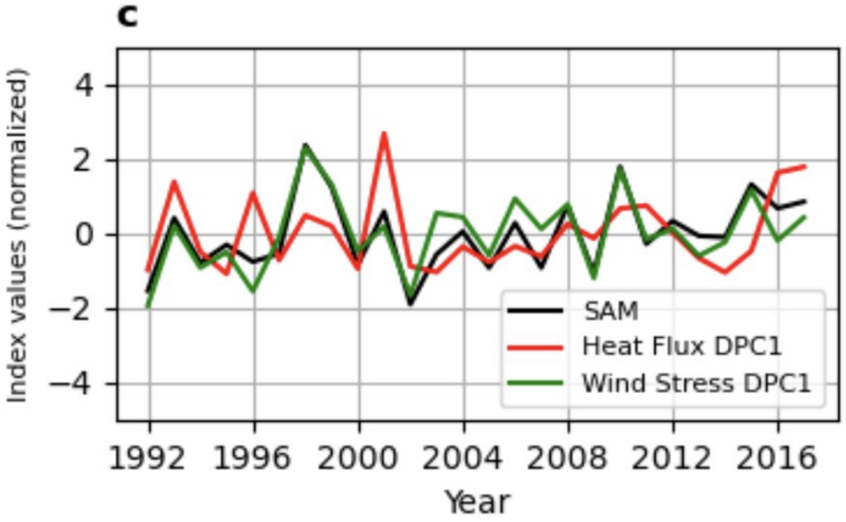


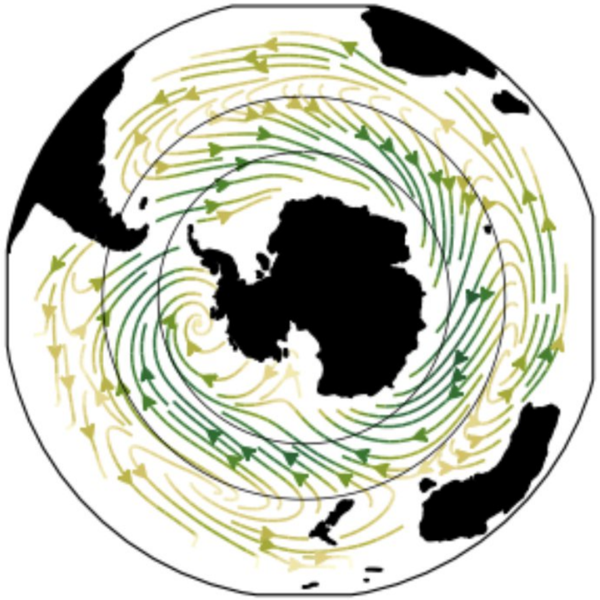
SAM regression onto wind stress



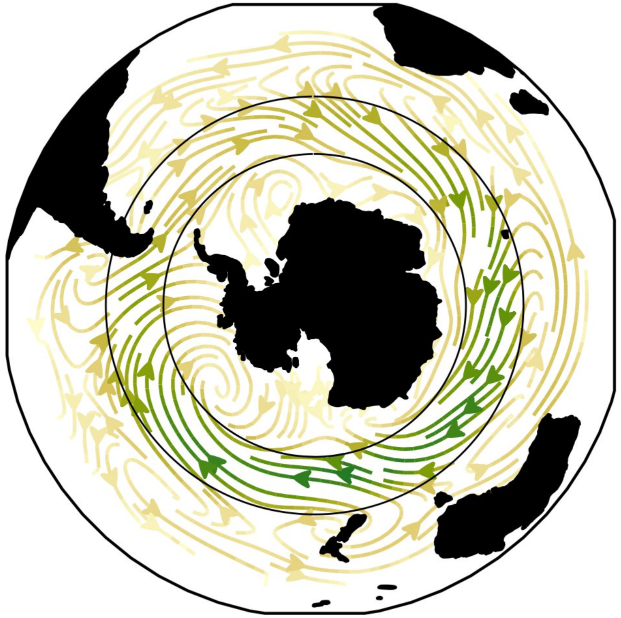
Leading EDF of wind stress



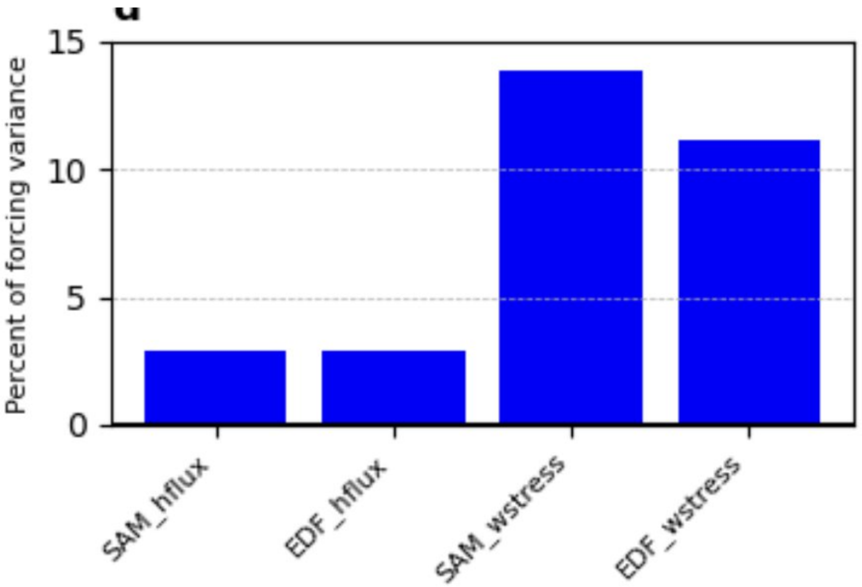
But, the EDF explains less overall variance in wind stress than the SAM, indicating their optimization for ocean variability.



SAM regression onto wind stress

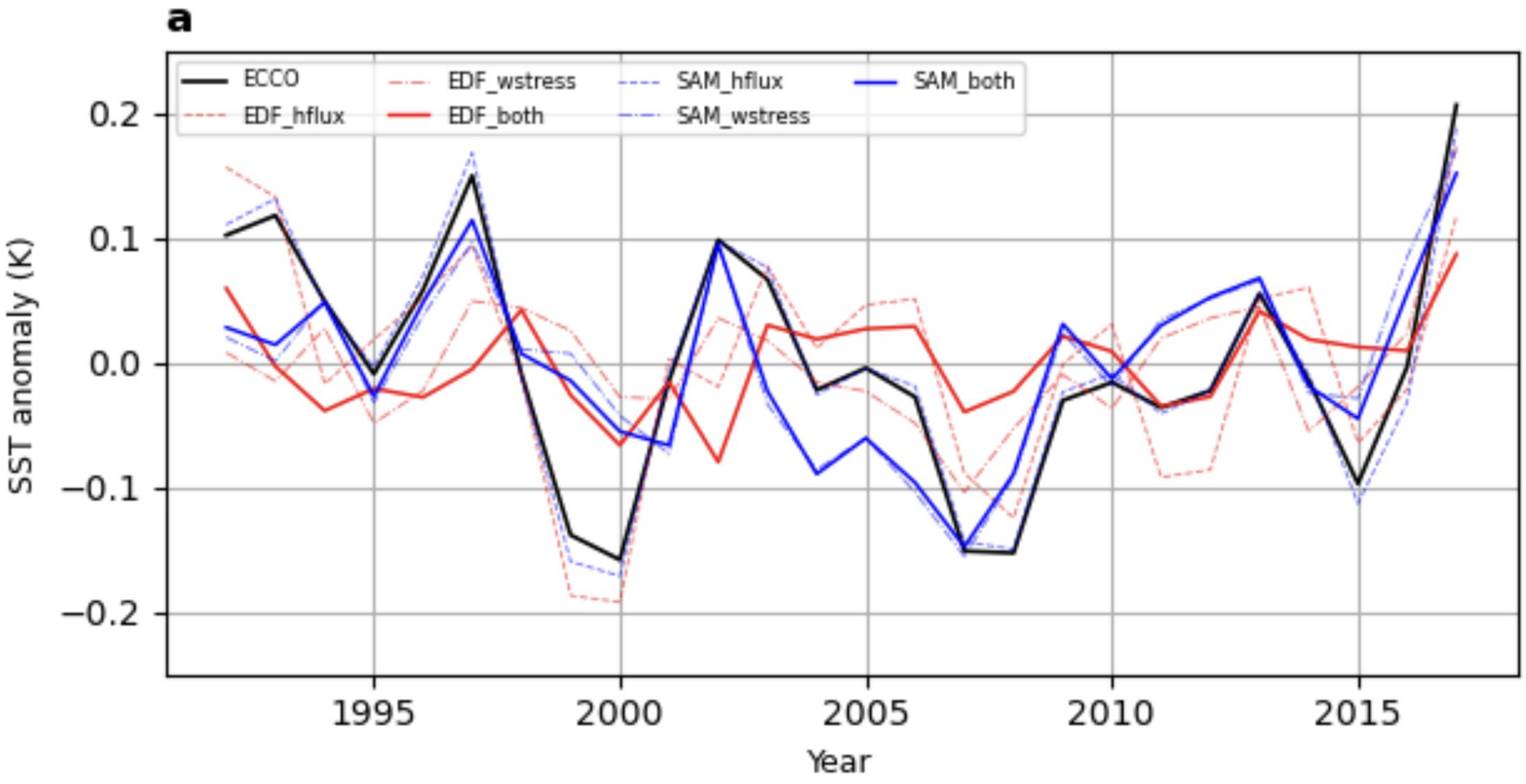


Leading EDF of wind stress



DPCAs give a parsimonious representation of the *linear* component of ocean variability. But do they have explanatory power in the context of the nonlinear ocean?

Modified ECCO simulations show that the derived patterns account for far more variability in $\overline{SST}$ than the SAM



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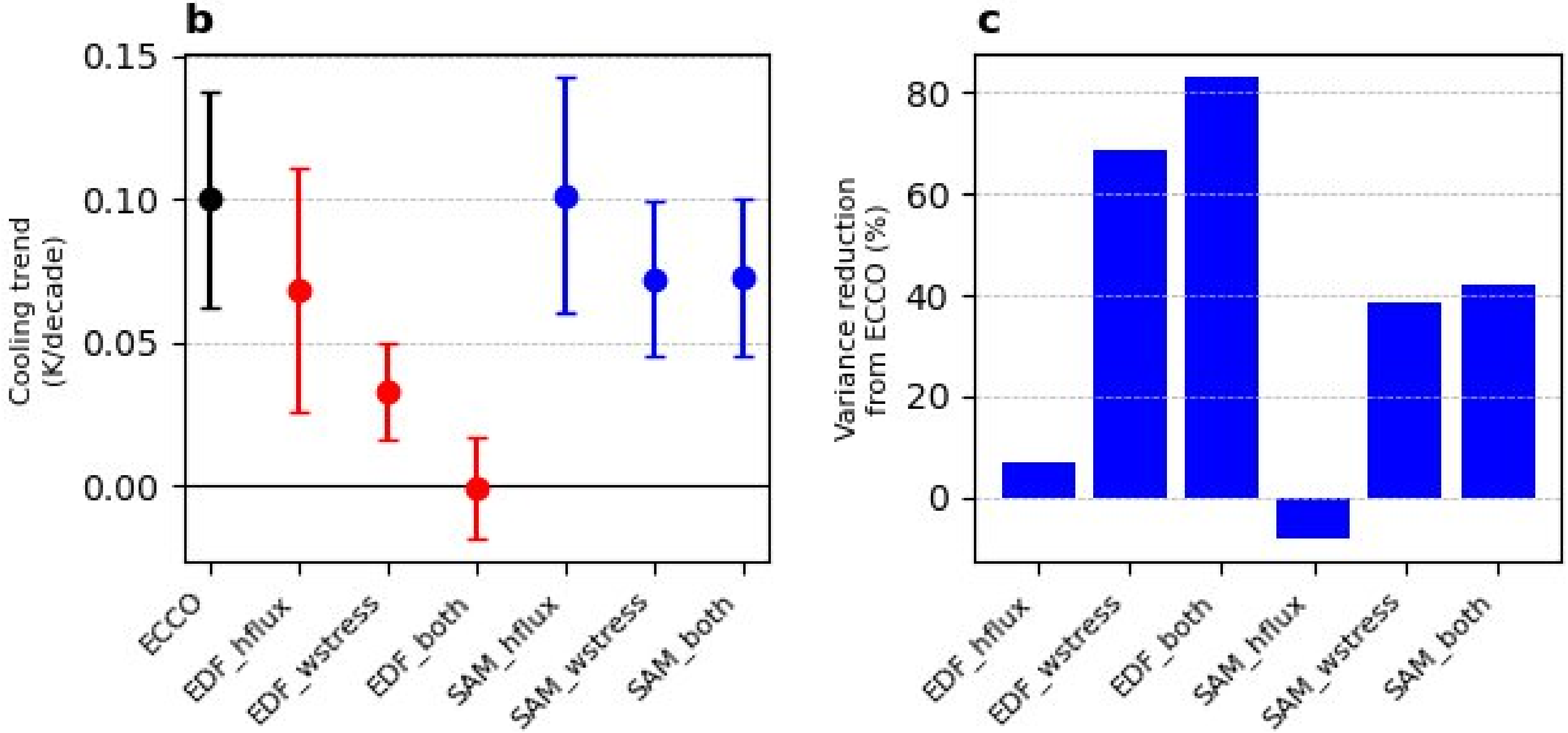


What does this show us? The Southern Ocean temperature signal can be driven efficiently by local wind stress patterns over the Pacific (this is where the cooling signal is most significant: Wills et al., 2022)

Ongoing work: Build out the adjoint framework to study zonally asymmetric variability

- Use as our QoI an index for differences in heat content between the Pacific sector and the rest of the Southern Ocean (Song et al., 2024): Provides a naturally low-pass filtered time series of Pacific variability
- **Hypothesis:** There exist atmospheric modes across variables (e.g. IPO) that can explain first-order variability independent of model controls
- Try to compute modes across variables: Are there drivers that extend beyond a single variable and drive the QoI?
- If so, can we explain variance in the state-forced model with sea ice and feedbacks enabled? Even though the modes are derived from flux variables?

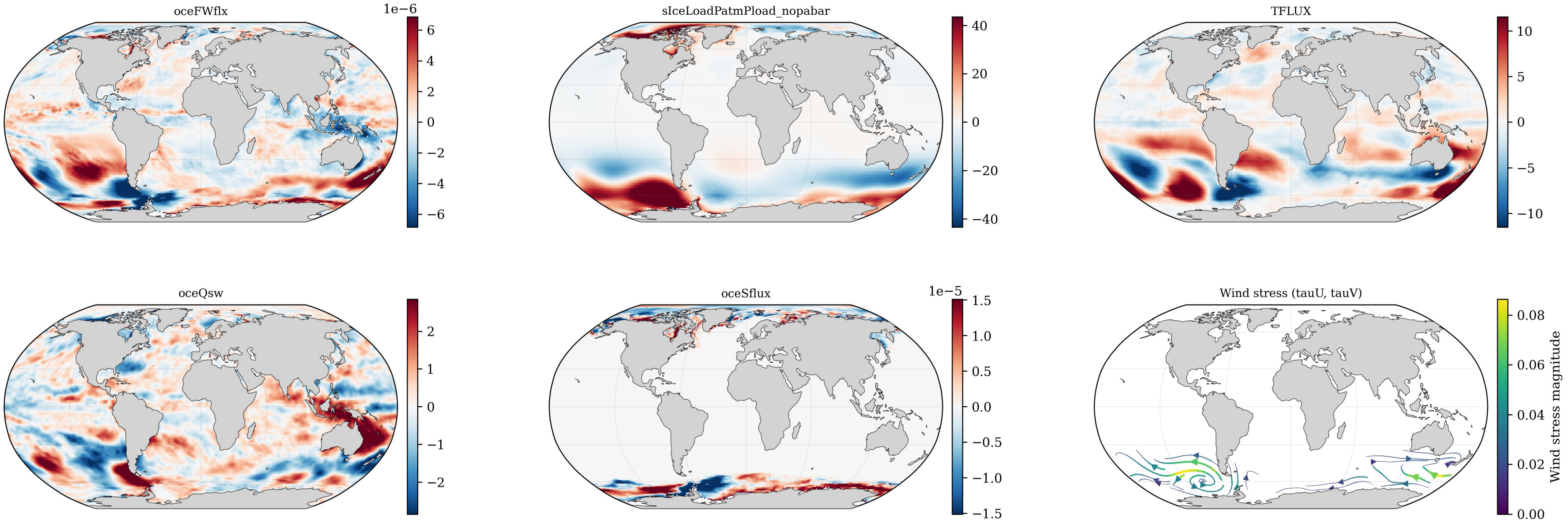
A bit more math

- Earlier, we computed the modes as the left singular vectors of the matrix $\left(\mathbf{S}^{\mathrm{T}}\boldsymbol{\phi}\right)$, where $\boldsymbol{\phi}$ is a single forcing variable.
- Now, we take the matrix $\sum_i \left(\mathbf{S}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{\phi}_{\boldsymbol{i}}\right)$, which normalizes all forcing variables by their corresponding adjoint sensitivities.
- Given the decomposition $\mathbf{U\Sigma V}^{\mathrm{T}} = \sum_i \left(\mathbf{S}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{\phi}_{\boldsymbol{i}}\right)$, we can now approximate the QoI using the leading singular vectors:

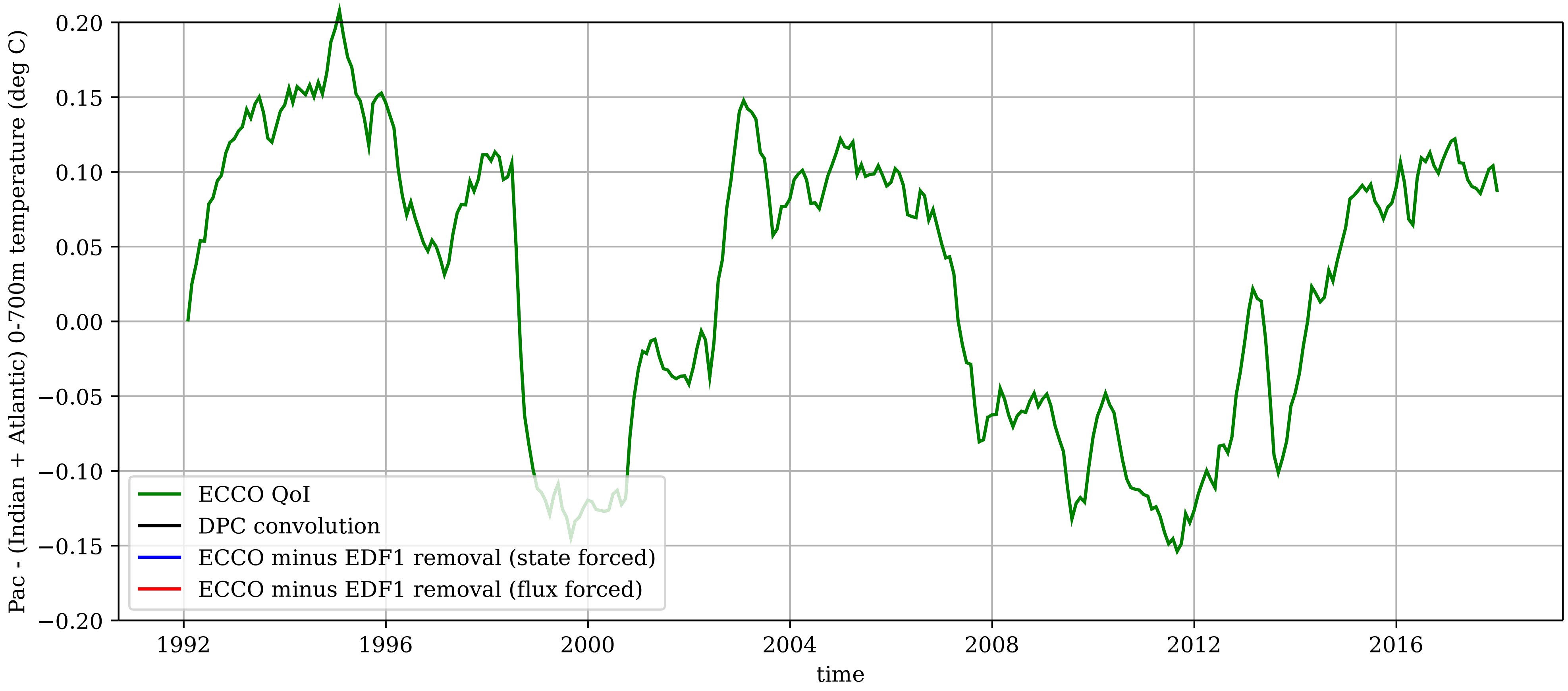
$$\delta J(t) \approx \sigma_1(\mathbf{u_1} * \mathbf{v_1})$$

(Fukumori et al., 2021: $\delta J(t) \approx \sum_i \sum_{\mathbf{r}} \sum_{\Delta t} \frac{\partial J}{\partial \phi_i}(\mathbf{r}, \Delta t)\Big|_{t_g} \delta\phi_i(\mathbf{r}, t - \Delta t).$)

Compound EDFs now yield patterns across all forcing patterns



Forward experiments validate linear approximation



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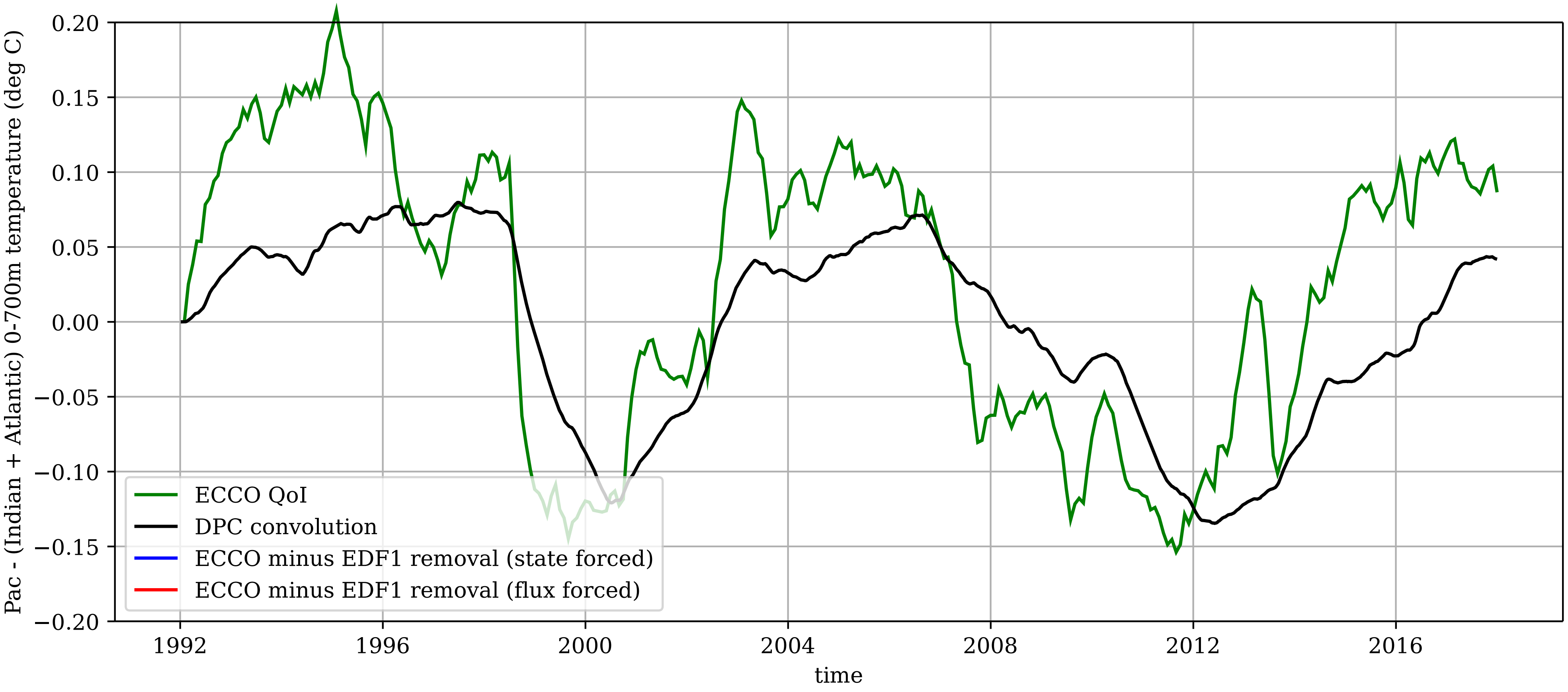

$$QoI(t) \approx \sigma_1(\mathbf{u_1} * \mathbf{v_1})$$

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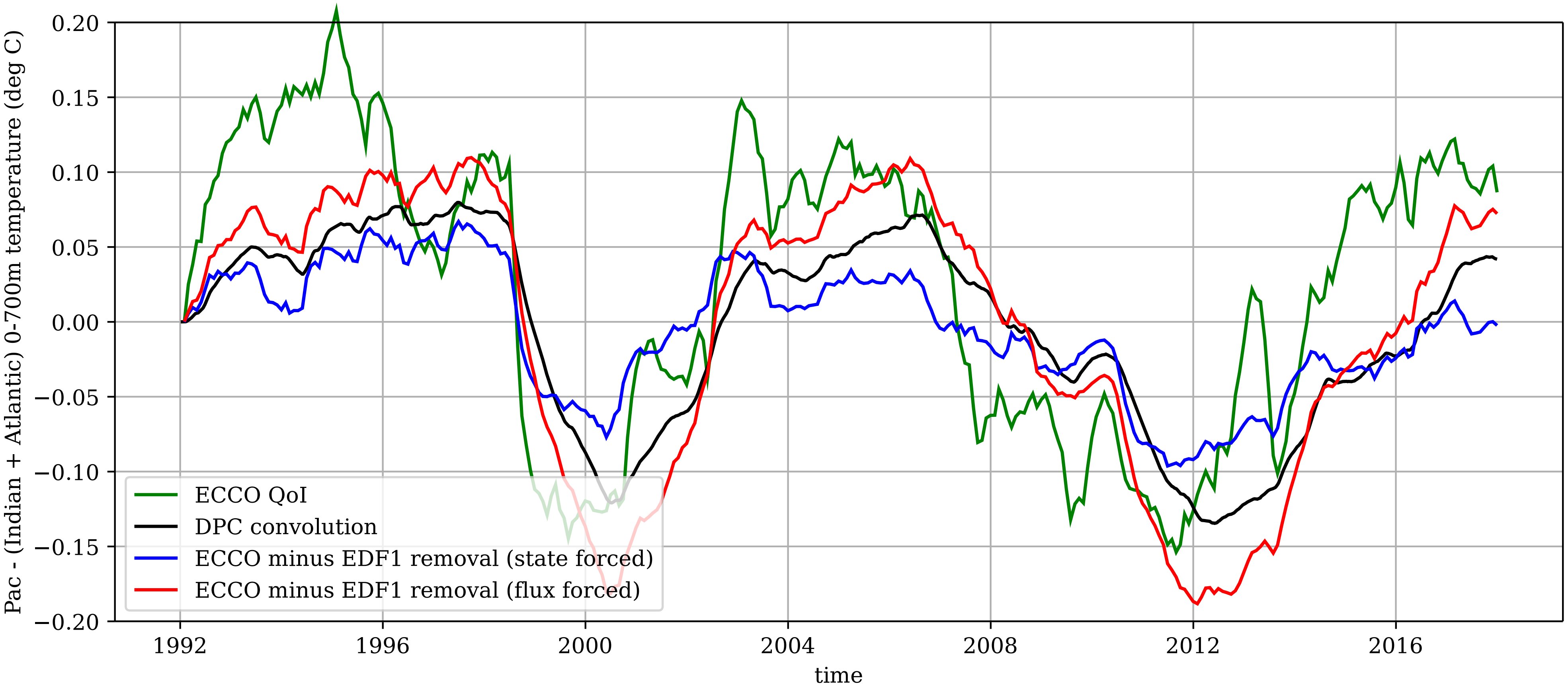

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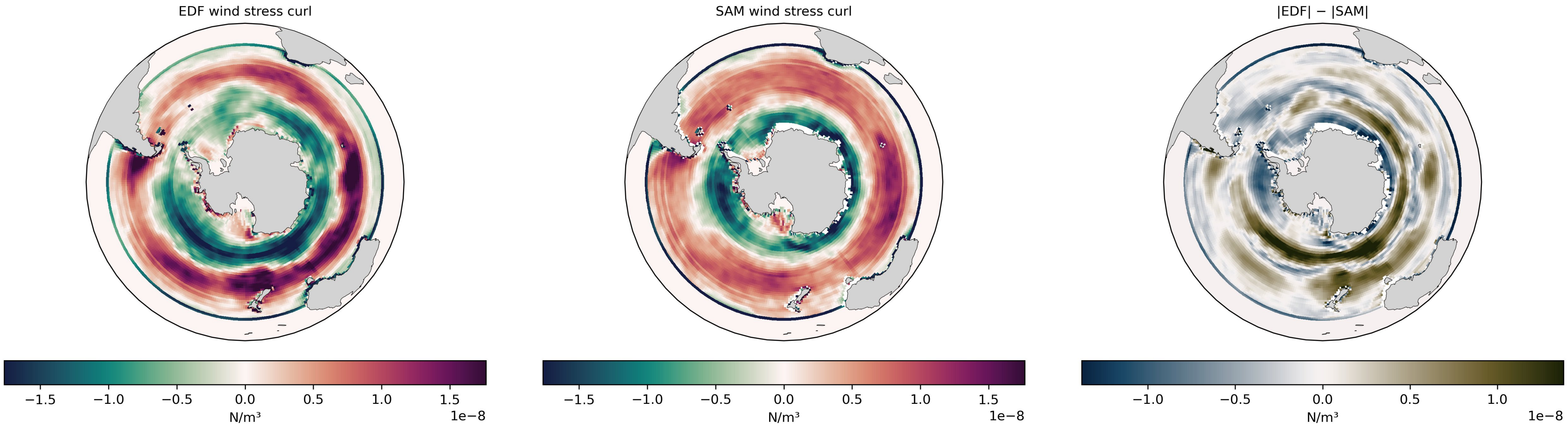
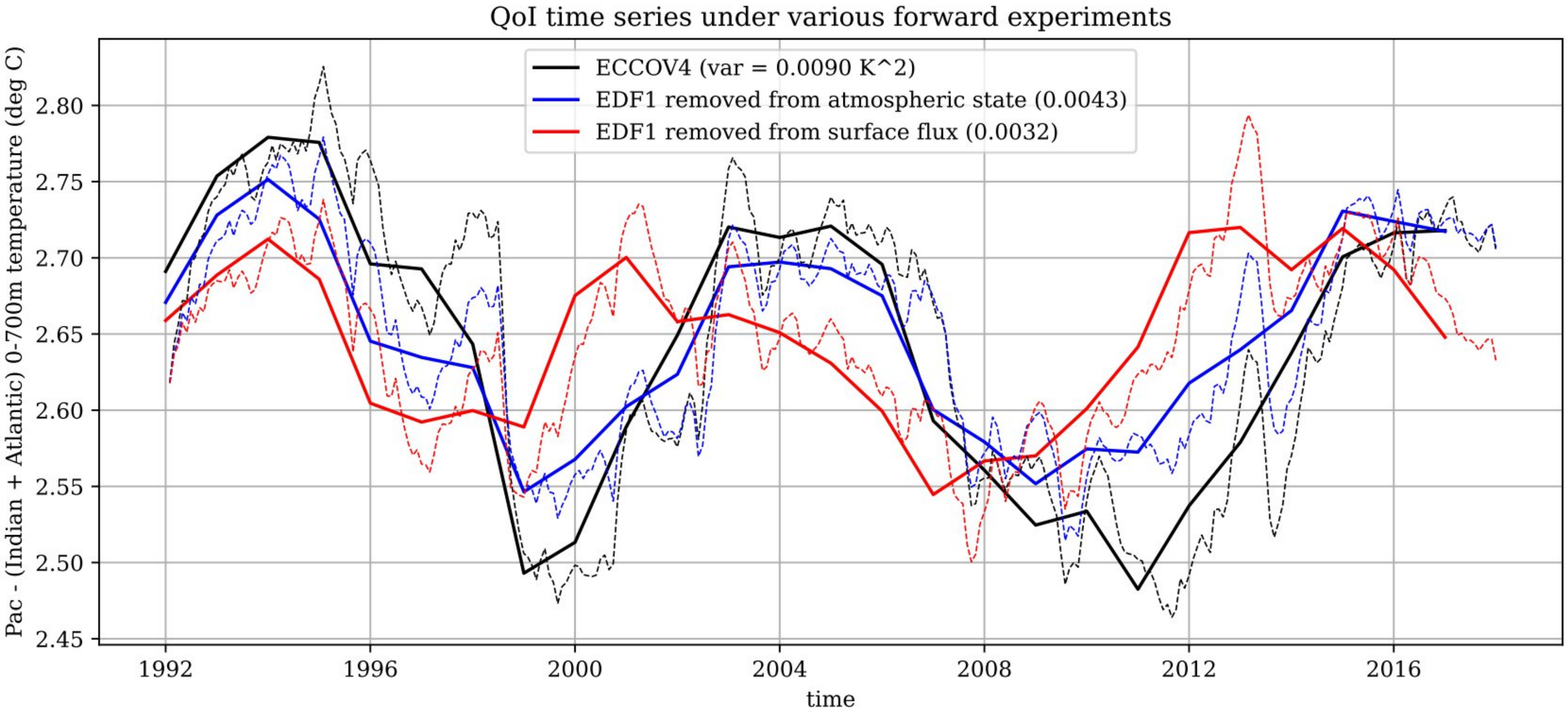
Takeaways + Next steps

- We have a statistical method that uses the adjoint to generate a parsimonious representation of ocean variability
- This can highlight pathways of variability that may be obscured by correlational studies or idealized modeling experiments
- If this sounds like it could be useful in your work, or you have ideas of caveats/extensions, please reach out! (I’m looking to continue modeling work in a postdoc)
- Watch my PhD defense July 16 for a more detailed story! (ocean.washington.edu)

Wind stress curl comparison




QoI time series under various forward experiments
ECCOV4 (var = 0.0090 K^2)
EDF1 removed from atmospheric state (0.0043)
EDF1 removed from surface flux (0.0032)
Pac - (Indian + Atlantic) 0-700m temperature (deg C)
2.80
2.75
2.70
2.65
2.60
2.55
2.50
2.45
1992
1996
2000
2004
2008
2012
2016
time