Tangent-linear modeling of underwater acoustics

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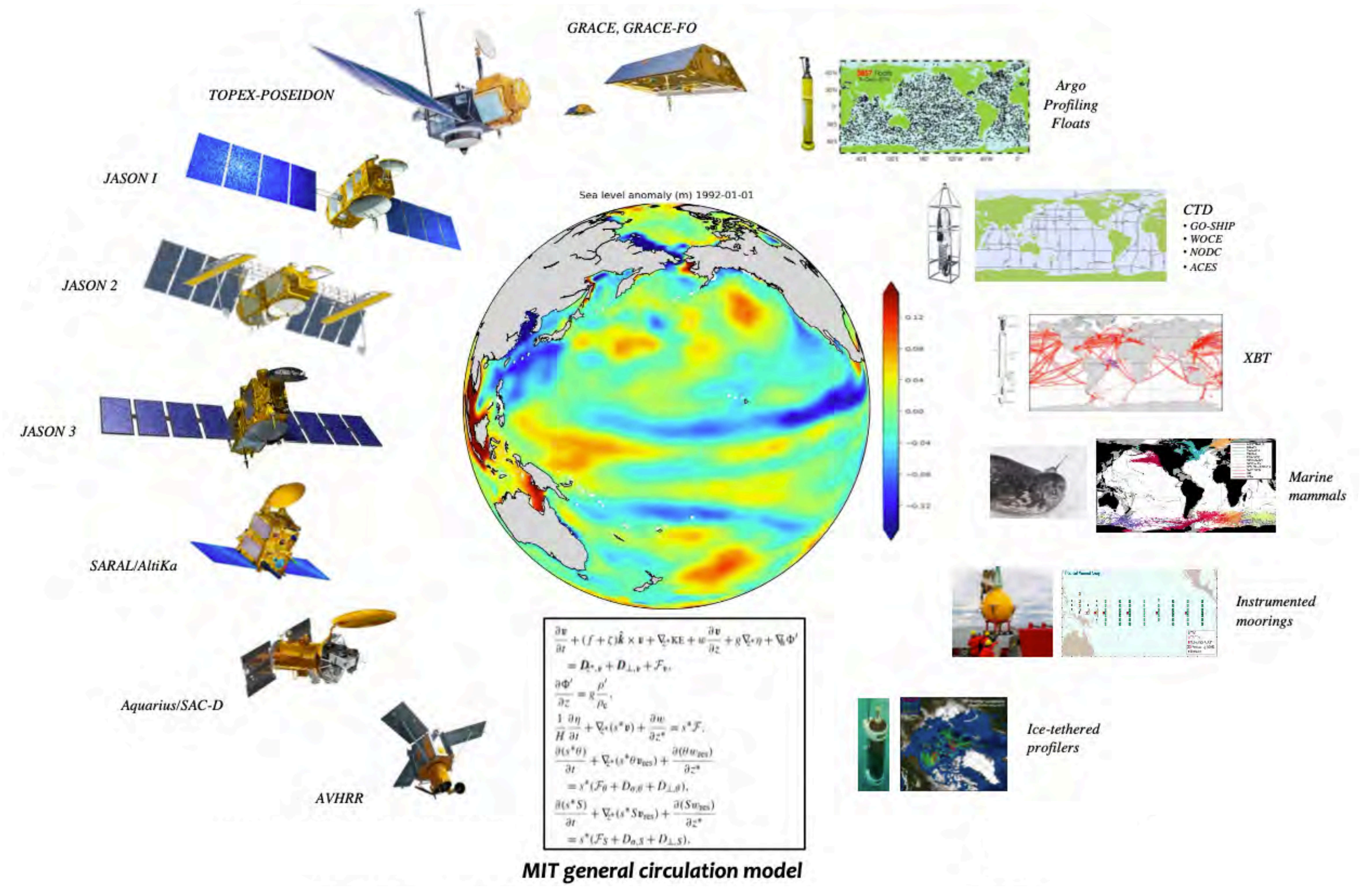
Ocean state estimation

Dynamically consistent ocean fields corrected with global observations

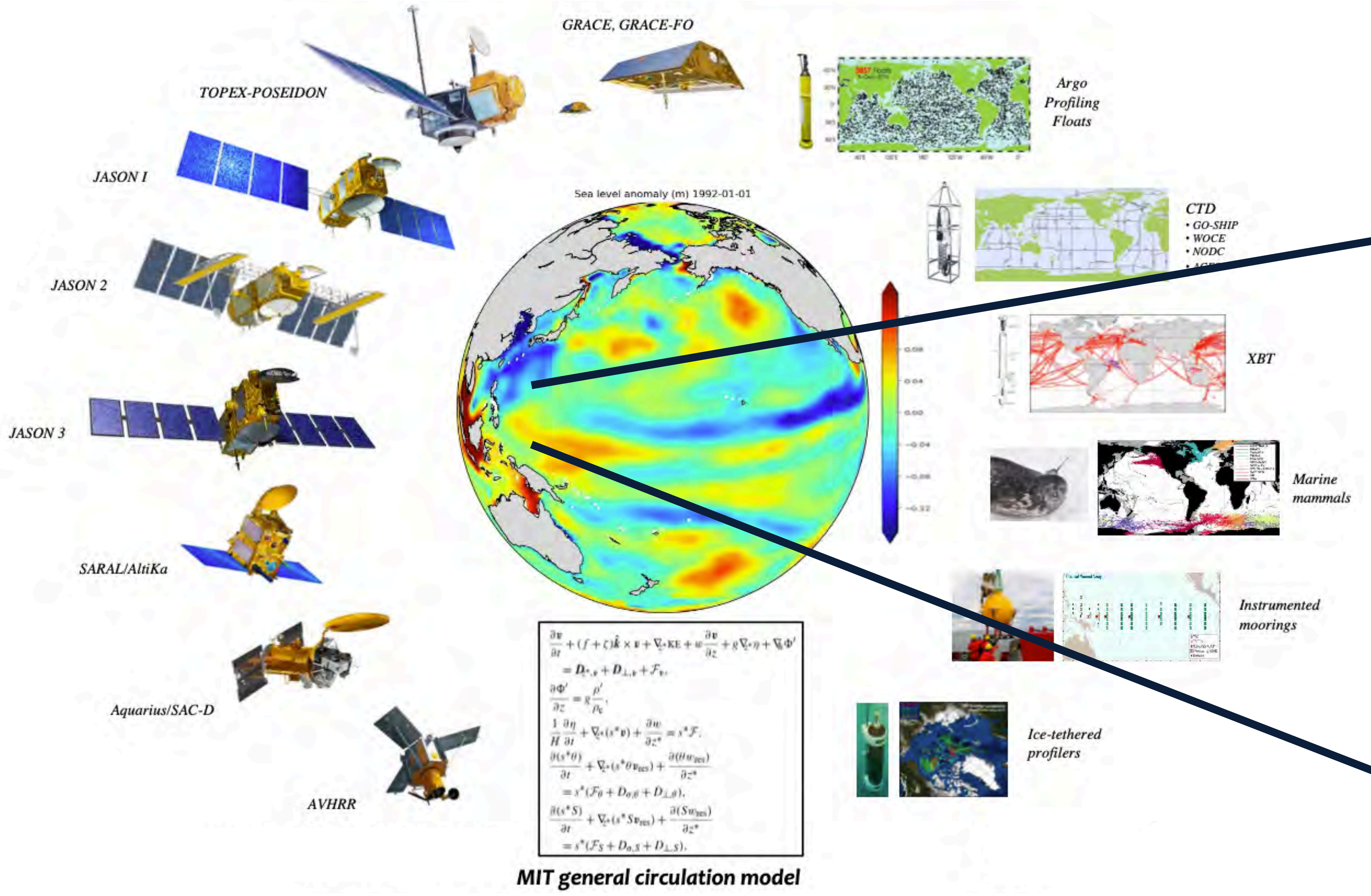


Source: Fenty, I *ECCO Meeting* (2018)

- **Data assimilation** of global ocean circulation synthesizes multiple decades via adjoint modeling
- Observing systems sparsely capture ocean >2 000 meters deep

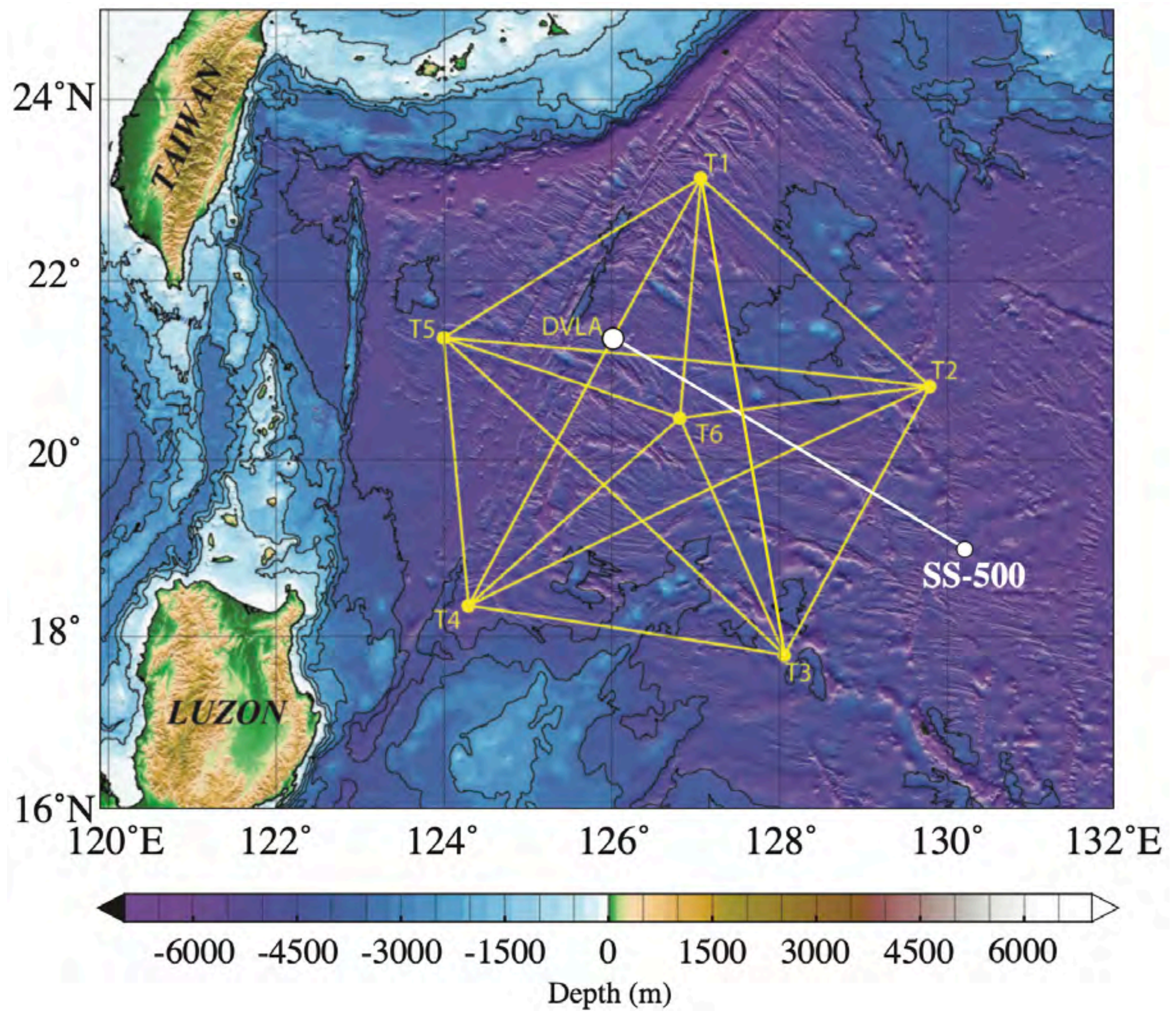
Ocean state estimation

Global observing system coverage is too sparse



Source: Fenty, I *ECCO Meeting* (2018)

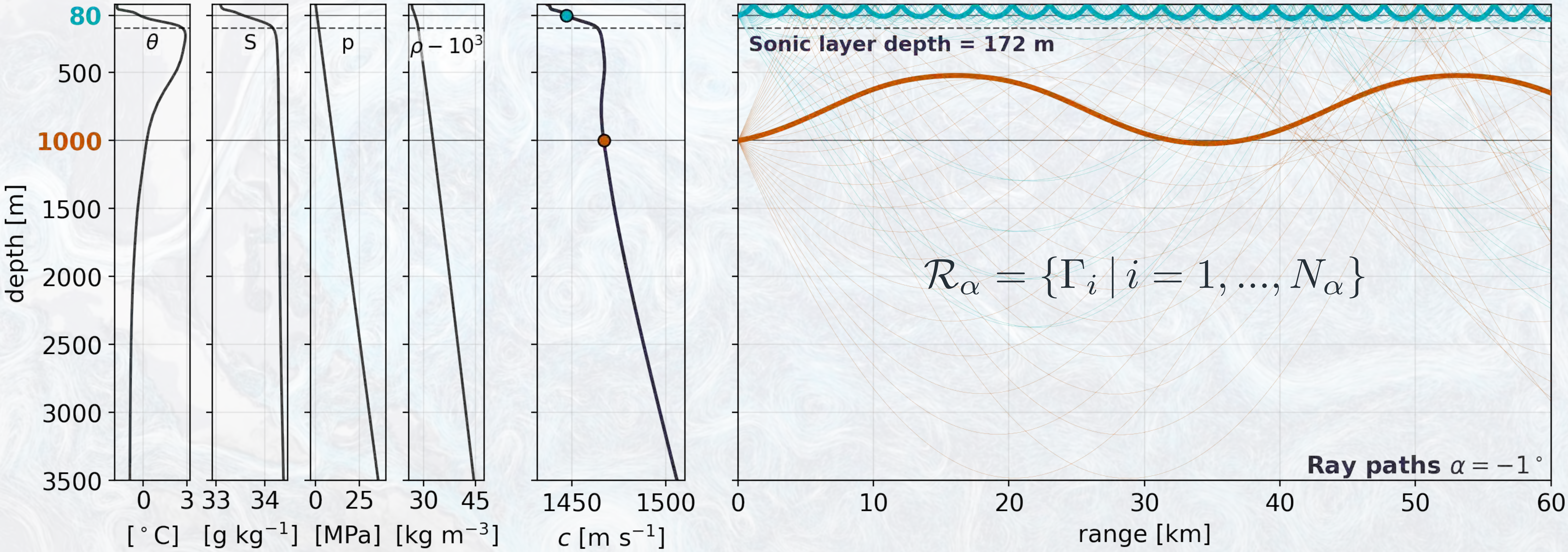
Include **underwater sound propagation** to the list of assimilated observing systems



Source: Gopalakrishnan *et al.* (2021)

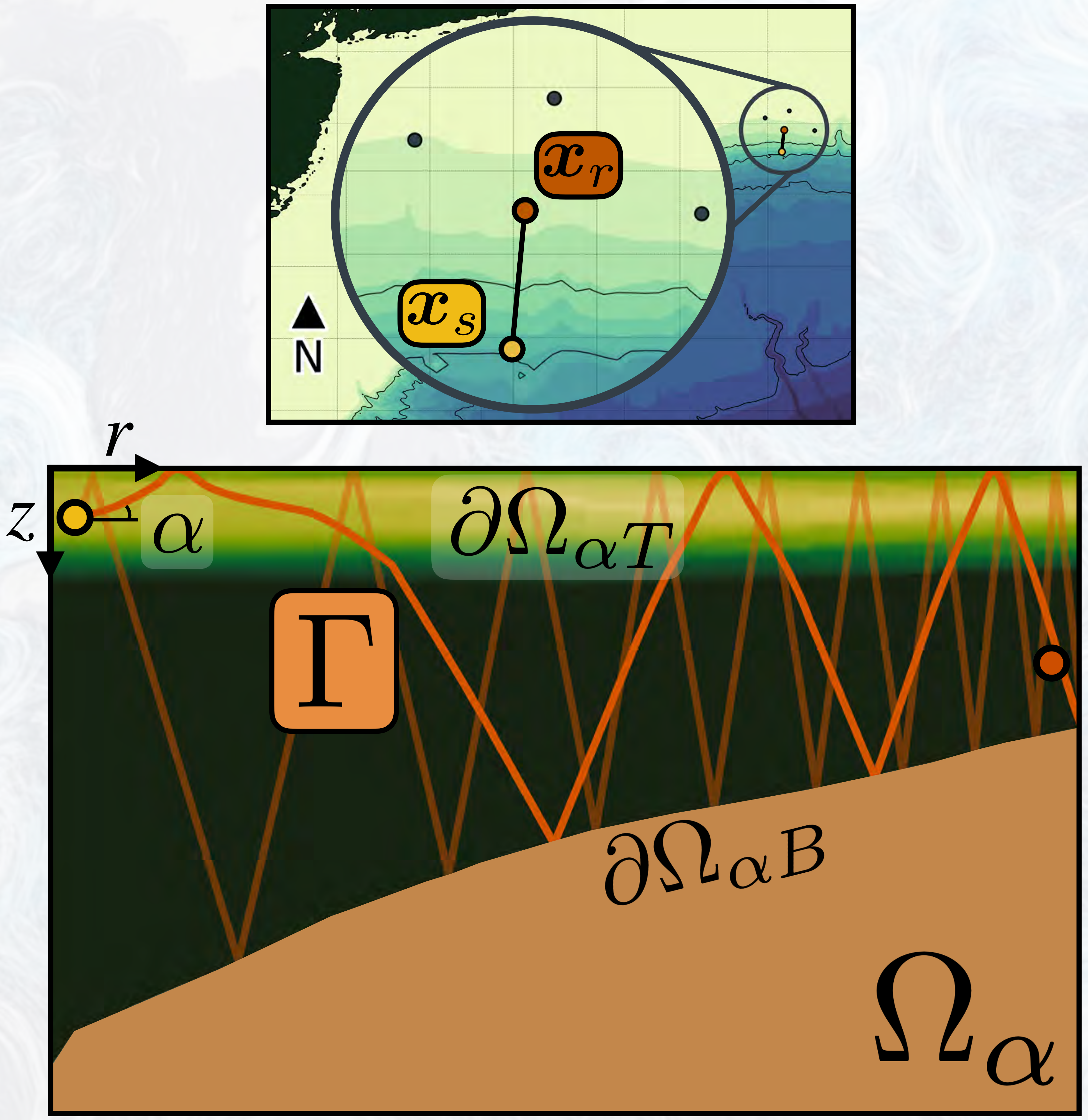
Sound propagation in oceans

Two-dimensional ray tracing underwater for two source depths



Underwater acoustic model

For a single ray path launched in range dependent media



With the acoustic state $\boldsymbol{a}(\gamma) = [\boldsymbol{r}, \tau, \boldsymbol{q}, \boldsymbol{\beta}]^T$,
let $f' = df/d\gamma$

$$
\begin{aligned}
\boldsymbol{r}'' &= -\nabla c/c & \qquad A &= |\mathcal{R}|A & \qquad &\partial\Omega_{\alpha B}\\
\tau' &= 1/c & \boldsymbol{r}' &= \boldsymbol{\mathcal{T}}_B & &\partial\Omega_{\alpha B}\\
\boldsymbol{q}' &= c\boldsymbol{\beta} & \boldsymbol{r}' &= \boldsymbol{\mathcal{T}}_T & &\partial\Omega_{\alpha T}\\
\boldsymbol{\beta}' &= -(\nabla^2 c/c^2)\boldsymbol{q} & \nabla\boldsymbol{r}\cdot\boldsymbol{n} &= 0 & &\partial\Omega_{\alpha R}\\
& & \boldsymbol{a} &= \boldsymbol{a}_0 & &\partial\Omega_{\alpha S}\\
& & \boldsymbol{r}' &= \boldsymbol{\mathcal{T}}_0 & &\partial\Omega_{\alpha S}
\end{aligned}
$$

Eigenray identification

Geometric beam spreading with hat-shaped decay

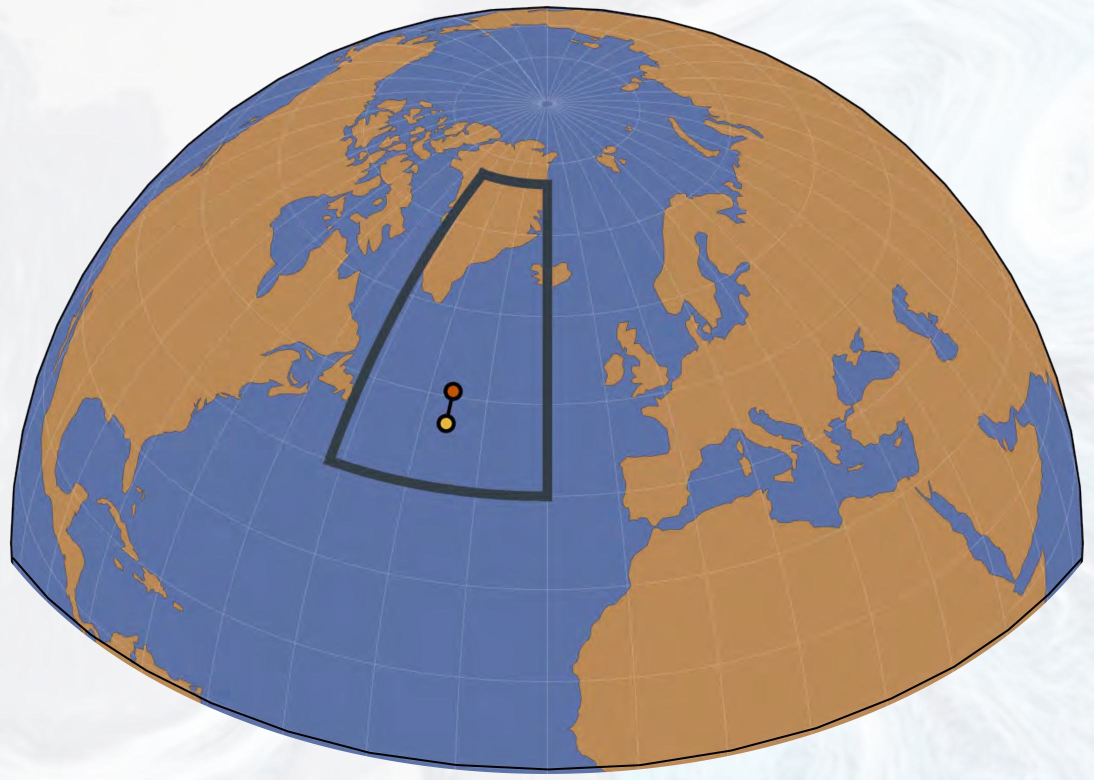
- A subset family of rays are found at a given receiver position $\boldsymbol{r}_r$ by smearing functions over the beam halfwidth $W(\gamma) = |q(\gamma)\delta\alpha|$,

$$\nu(\gamma) = \begin{cases} \frac{W(\gamma) - |\boldsymbol{n}(\gamma)\cdot\boldsymbol{r}_r|}{W(\gamma)} & \text{for } |\boldsymbol{n}\cdot\boldsymbol{r}_r| \leq W, \\ 0 & \text{otherwise} \end{cases}$$

- Tapered pressure amplitude is found for each eigenray arrival, $A_{\mathrm{arr}} = A\nu$

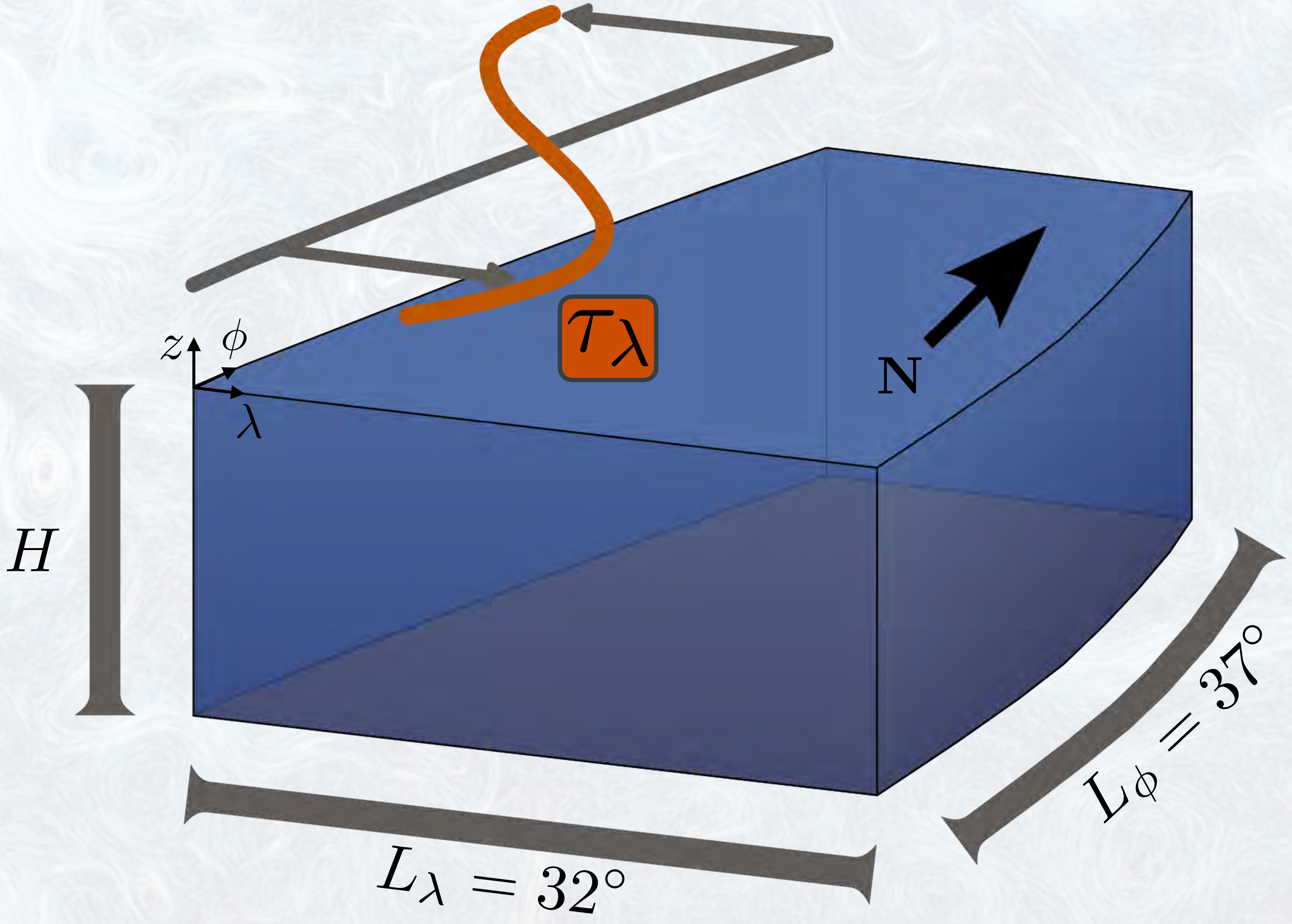
Subpolar gyre basin

Idealized single gyre with 100 discrete depth levels



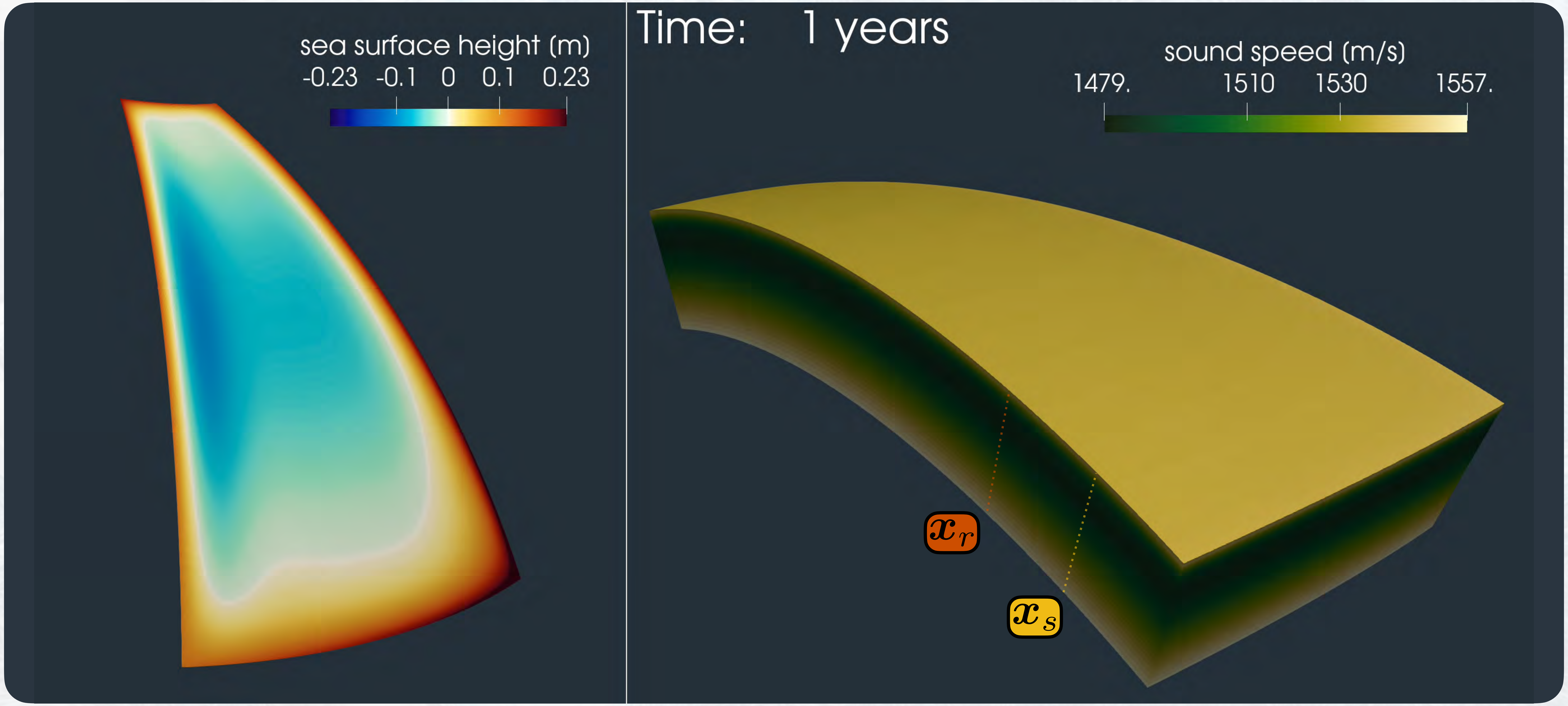
$$H = 6000\,\mathrm{m}$$

$$\Delta\lambda = \Delta\phi = \frac{1}{12}^{\circ}$$



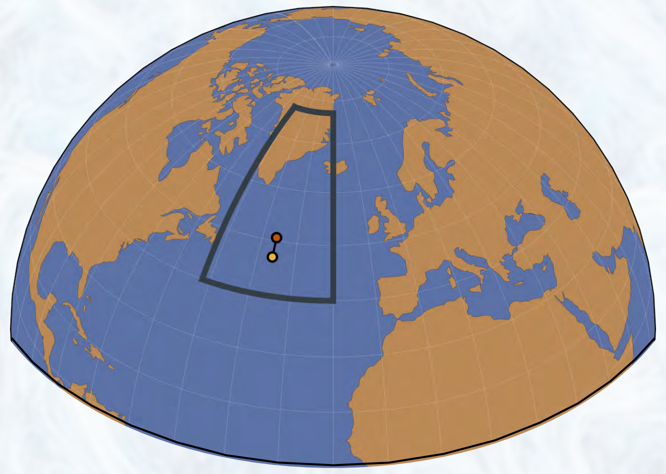
Ocean spinup evolves sound speed field

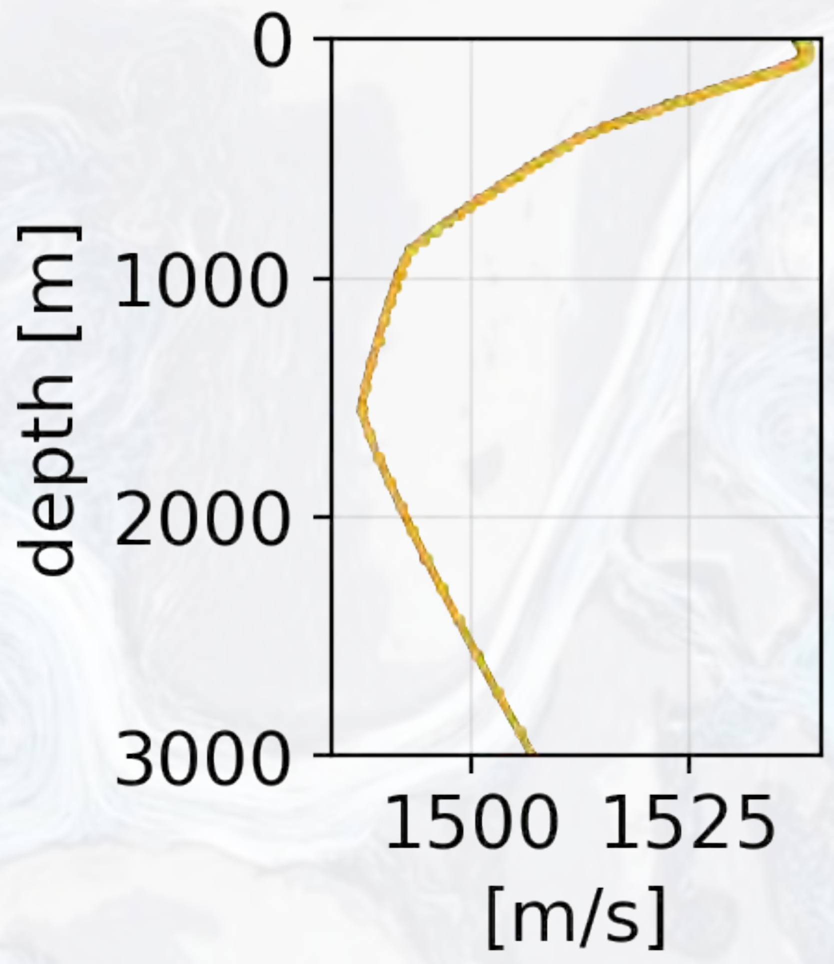
Yearly snapshots for one century, meridional acoustic geometry

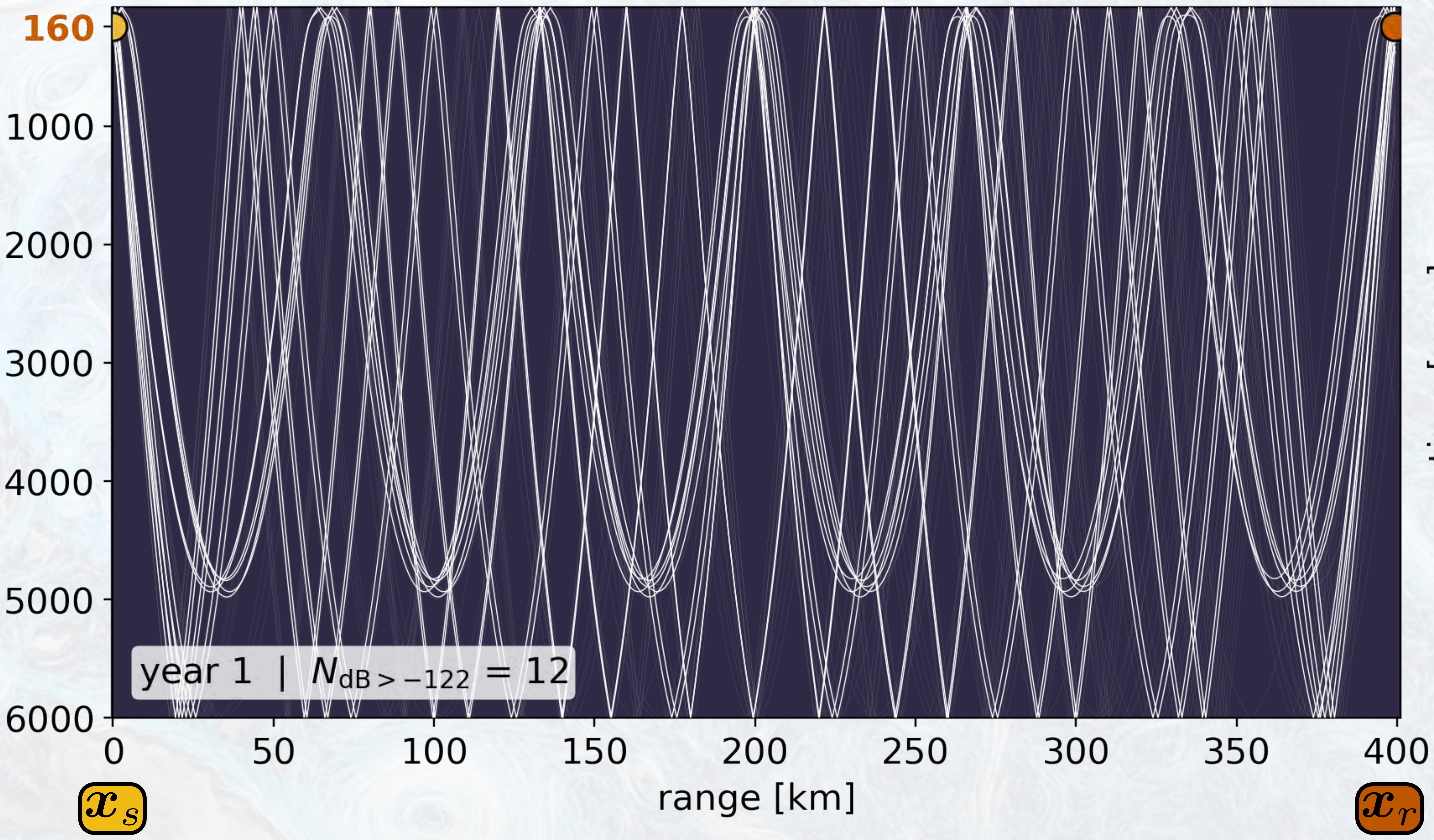


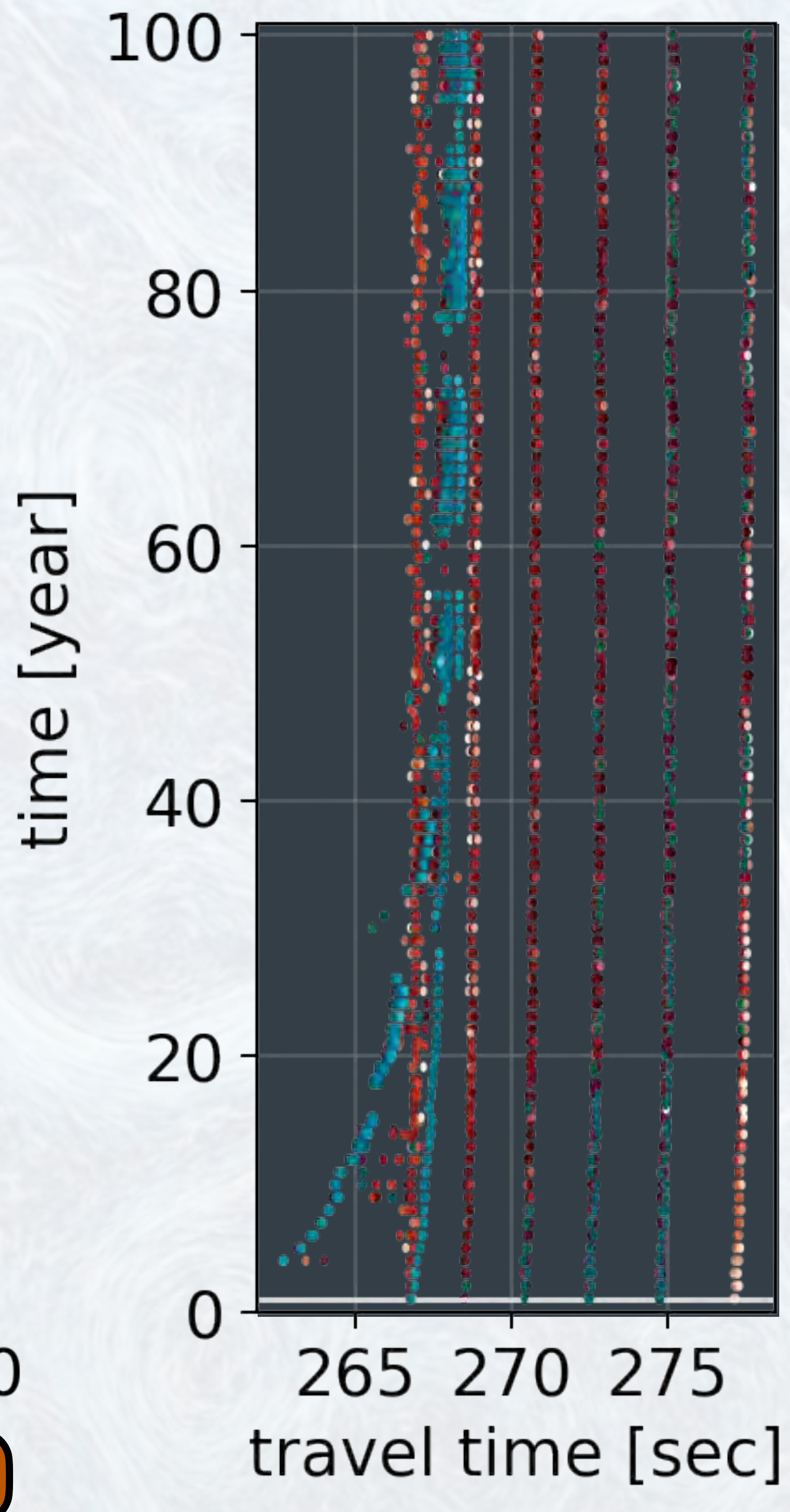
Subpolar gyre acoustic structure

Threshold eigenrays oscillate over multi-decadal cycles









Computing derivatives with AD

Solving discrete forward responses to perturbations

- Forward algorithmic differentiation (**AD**) solves both the forward model and increments of, J, the quantity of interest with respect to, $\boldsymbol{m}$, control parameters

$$\delta J = \nabla_{\boldsymbol{m}} J \cdot \delta \boldsymbol{m}$$

- Sensitivities are responses scaled along the directional derivative defined by a tangent linear model (**TLM**). Check performance of acoustic equations,

$$\underbrace{\boldsymbol{M}}_{\text{total TLM}} = \underbrace{\boldsymbol{B}}_{\text{acoustic}} \circ \underbrace{\boldsymbol{O}}_{\text{ocean}}$$

Computing derivatives with AD

Assessing performance according to ocean temperature perturbations

$$\boldsymbol{m} = \boldsymbol{\theta}_0(\boldsymbol{x})$$

- The initial temperature field directly impacts the acoustic state

$$\Delta\boldsymbol{\theta}_0(\boldsymbol{x}) = \epsilon \boldsymbol{P}$$

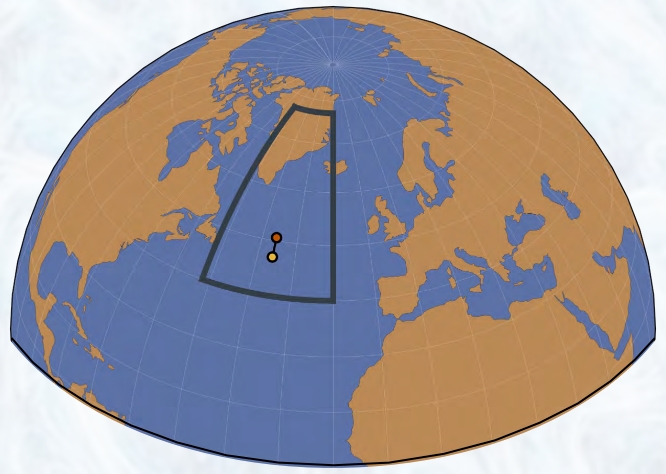
- Perturbations, $\boldsymbol{P}$, are prescribed on
  - A single basis direction
  - Three-dimensional structures

$$\epsilon = 1$$

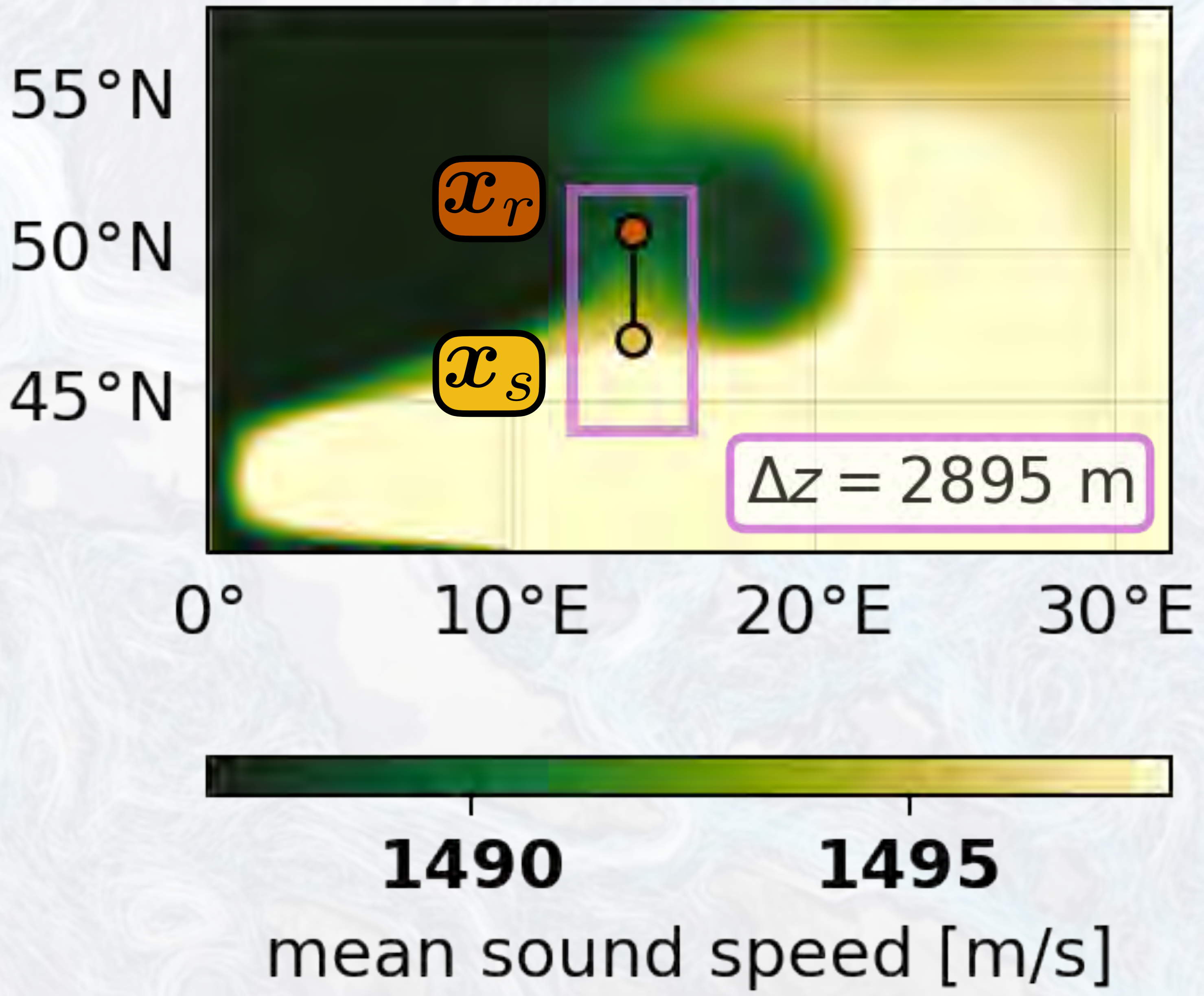
- Unitary scaling defines directional derivatives

BoxSea subdomain in subpolar gyre

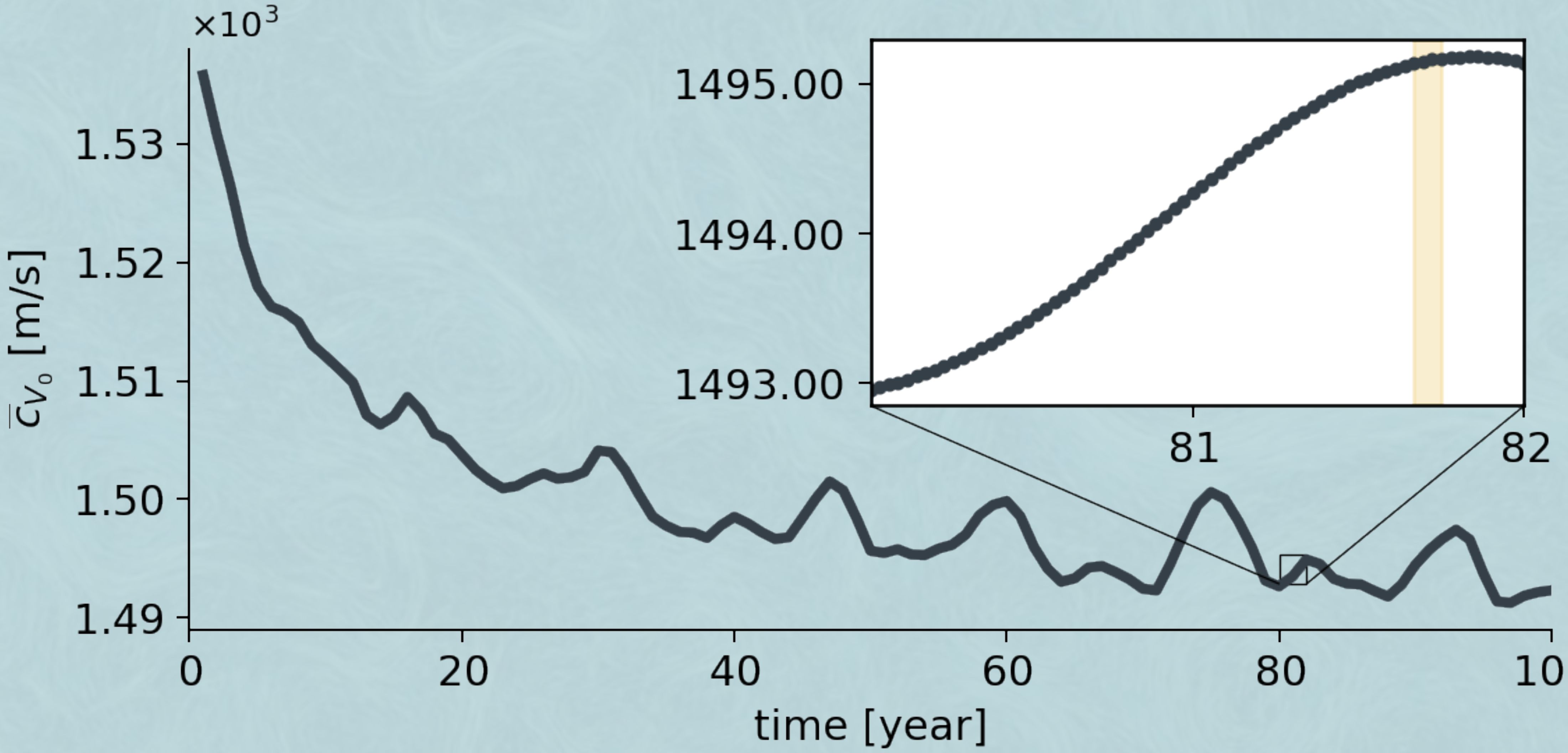
QoI of one month mean sound speed



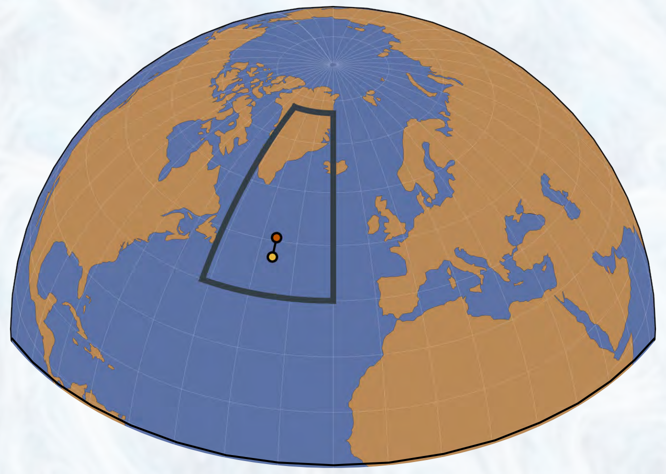
$$|\Omega_{\text{box}}| \sim \mathcal{O}(10^5)$$



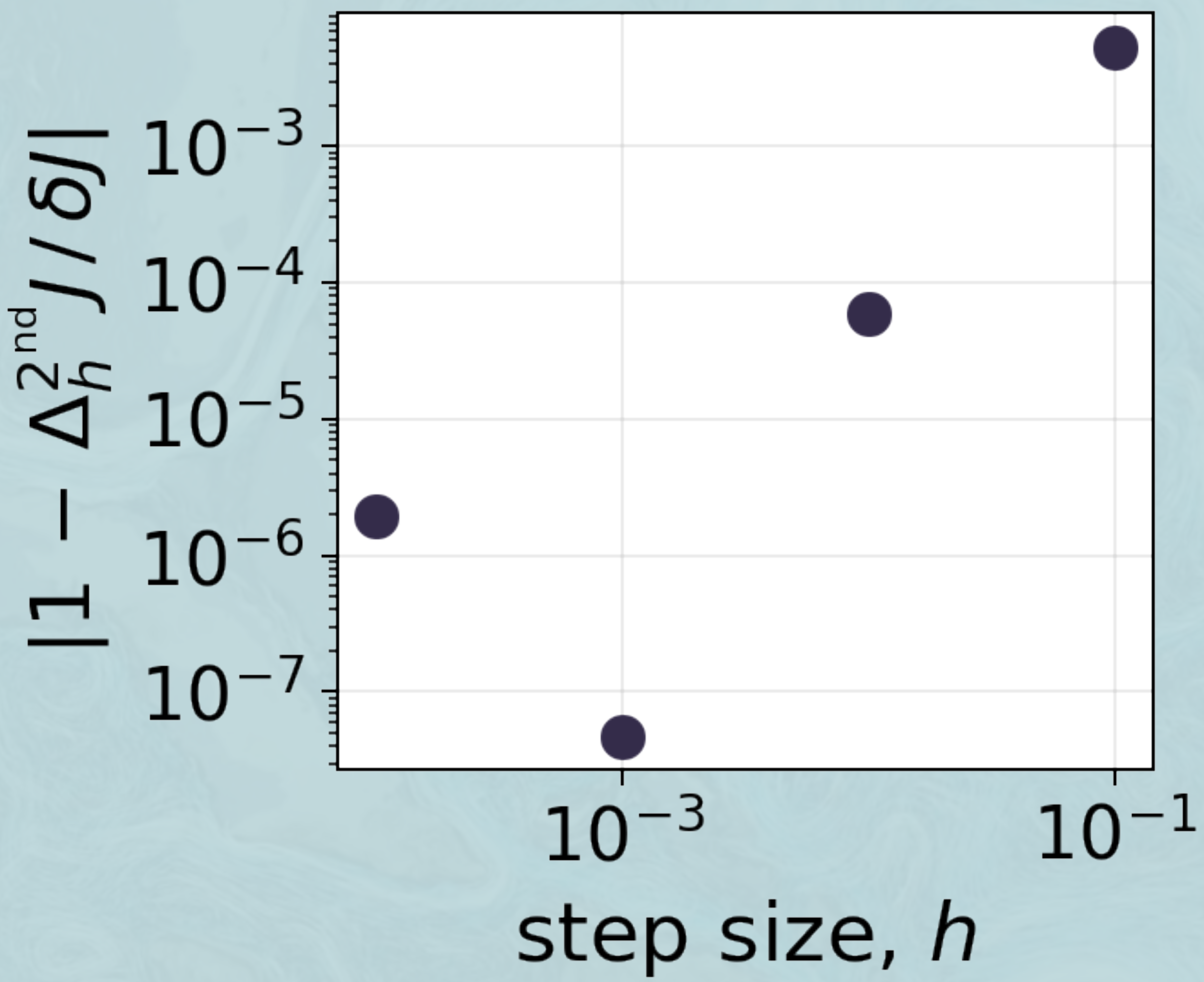
$$\bar{c} = \frac{1}{|\Omega_{\text{box}}|} \int_{\Omega_{\text{box}}} c(\boldsymbol{x}, t)\, dV$$



TLM increment in direction of BoxSea



Positive directional derivative for temperature changes



The initial temperature field creates a positive response to the mean sound speed field increment

Reference QoI: $J = 8955.2245\ \mathrm{m\,s^{-1}}$	
Case	$\delta J\ [\mathrm{m\,s^{-1}}]$
TLM	14.8646
FD, $h = 10^{-1}$	14.7845
FD, $h = 10^{-2}$	14.8637
FD, $h = 10^{-3}$	14.8646
FD, $h = 10^{-4}$	14.8646

Data assimilation updates for acoustics

ECCO's strong constrained 4DVar deterministic inversion

$$J(\boldsymbol{s}, \boldsymbol{m}) := \frac{1}{2} \sum_{t=t_1}^{t_f} \|\boldsymbol{F}\boldsymbol{m}(t) - \boldsymbol{y}_{\text{obs}}(t)\|^2_{\boldsymbol{R}(t)^{-1}}$$

- Discrete model-data misfit applies the parameter to observable (**pto**) map, $\boldsymbol{F}$, to model control parameters

$$\delta_{\boldsymbol{m}} J = \sum_{t=t_1}^{t_f} \delta\boldsymbol{m}(t)^T \boldsymbol{F}(t)^T \boldsymbol{R}(t)^{-1} \left[\boldsymbol{F}(t)\boldsymbol{m}(t) - \boldsymbol{y}_{\text{obs}}(t)\right]$$

- Variation to the control parameters define directional derivatives

$$J = \frac{1}{2} \sum_{t=t_1}^{t_f} \|\boldsymbol{E}\boldsymbol{\tau}(t) - \tau_{\text{obs}}(t)\|^2$$

- With $\boldsymbol{R} = \boldsymbol{I}$, a new pto map introduces comparison to observed **travel times**

Extend misfit for acoustics

pto map extracts eigenray travel times

$$\boldsymbol{m} = \boldsymbol{\theta}_0(\boldsymbol{x})$$

$$\boldsymbol{s}(t_1) = \boldsymbol{M}(\boldsymbol{\theta}_0), \quad \boldsymbol{s}(t_{i+1}) = \boldsymbol{M}(\boldsymbol{s}(t_i))$$

$$\boldsymbol{F}(t) = \underbrace{\boldsymbol{E}}_{\text{observable map}} \circ \underbrace{\boldsymbol{B}}_{\text{eigenrays}} \circ \underbrace{\boldsymbol{M}(t)}_{\text{ocean-acoustics}}$$

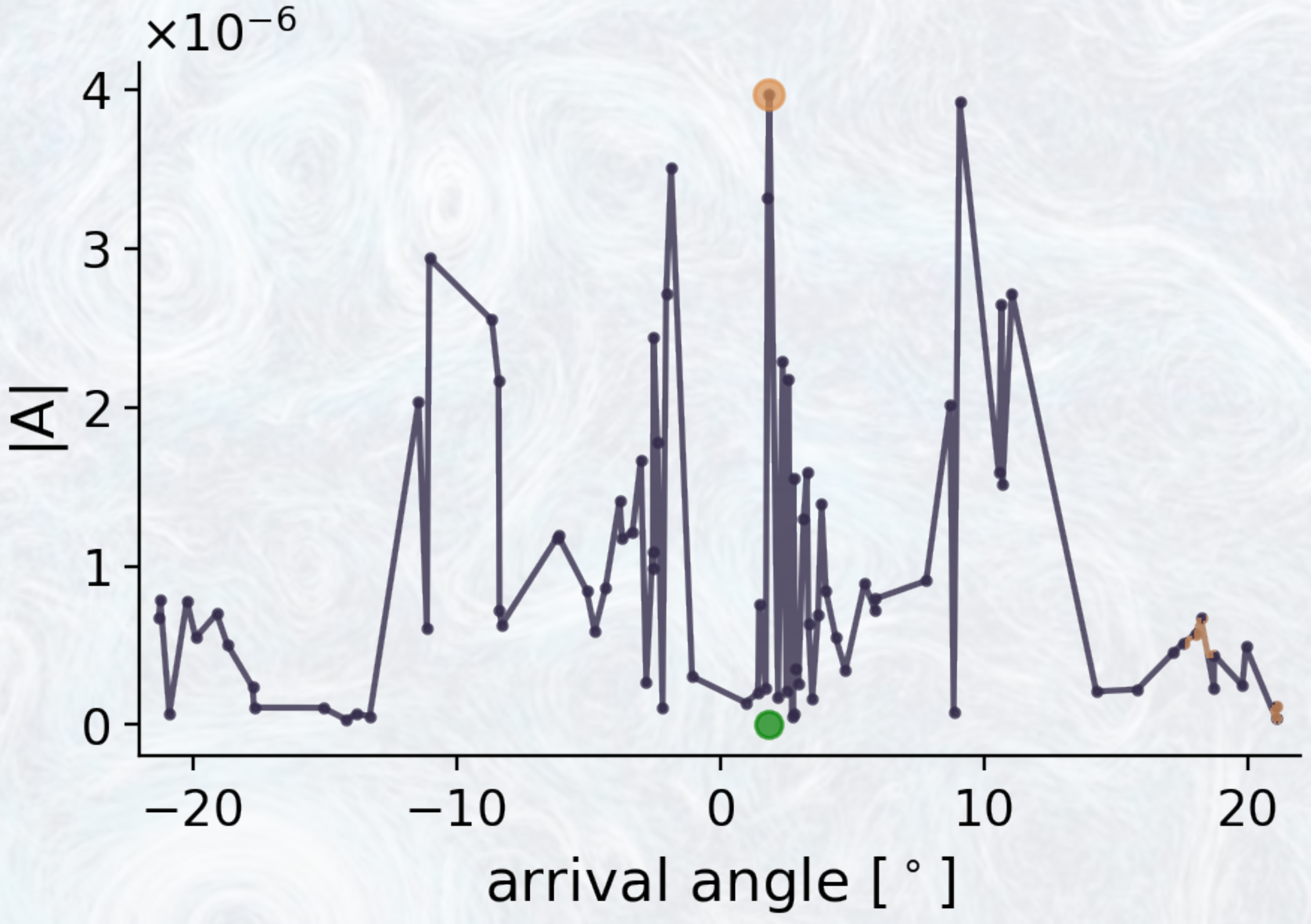
- Recall, the control parameter is potential temperature at initial time
- Ocean-acoustic state variables are evaluated with the coupled model $\boldsymbol{s} = [\boldsymbol{s},\, \boldsymbol{a}]^T$
- The acoustic pto finds the set subfamily of eigenrays and maps them to data space

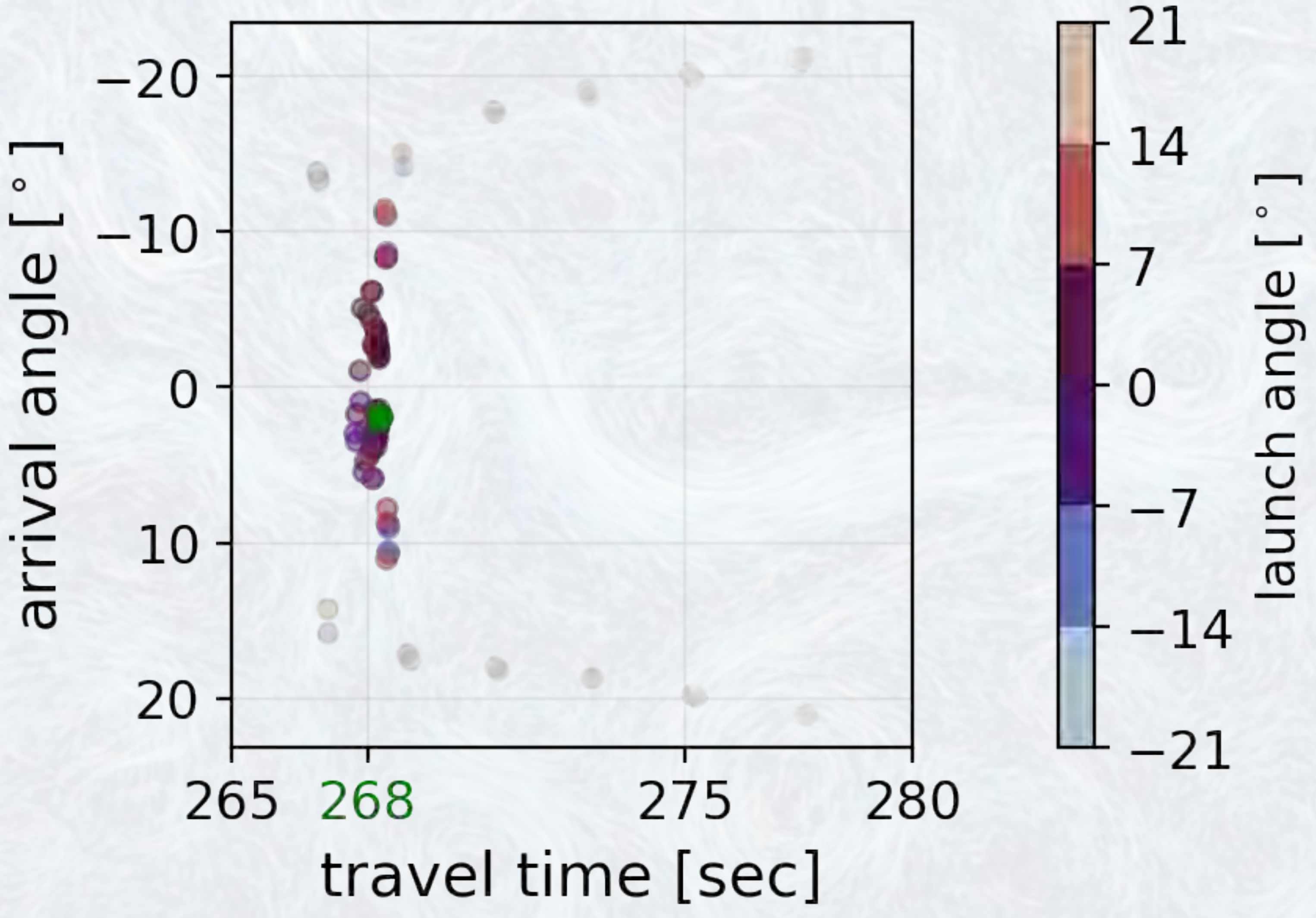
Acoustic observable map design

Treating observations as travel times from loudest arrivals

For a single source-receiver pair, N_e predicted eigenrays are found for each sound transmission. The strongest eigenray is defined as the ray with the largest acoustic pressure magnitude at the receiver range

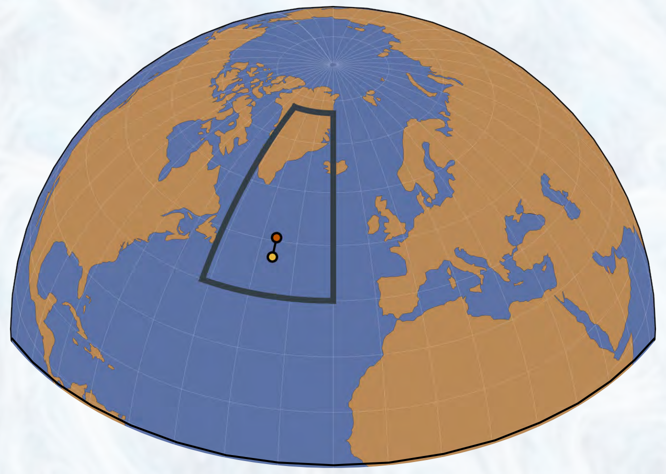
$$\boldsymbol{E}\boldsymbol{\tau}(t) = \boldsymbol{\tau}_{\arg\max(|A|)}(t)$$





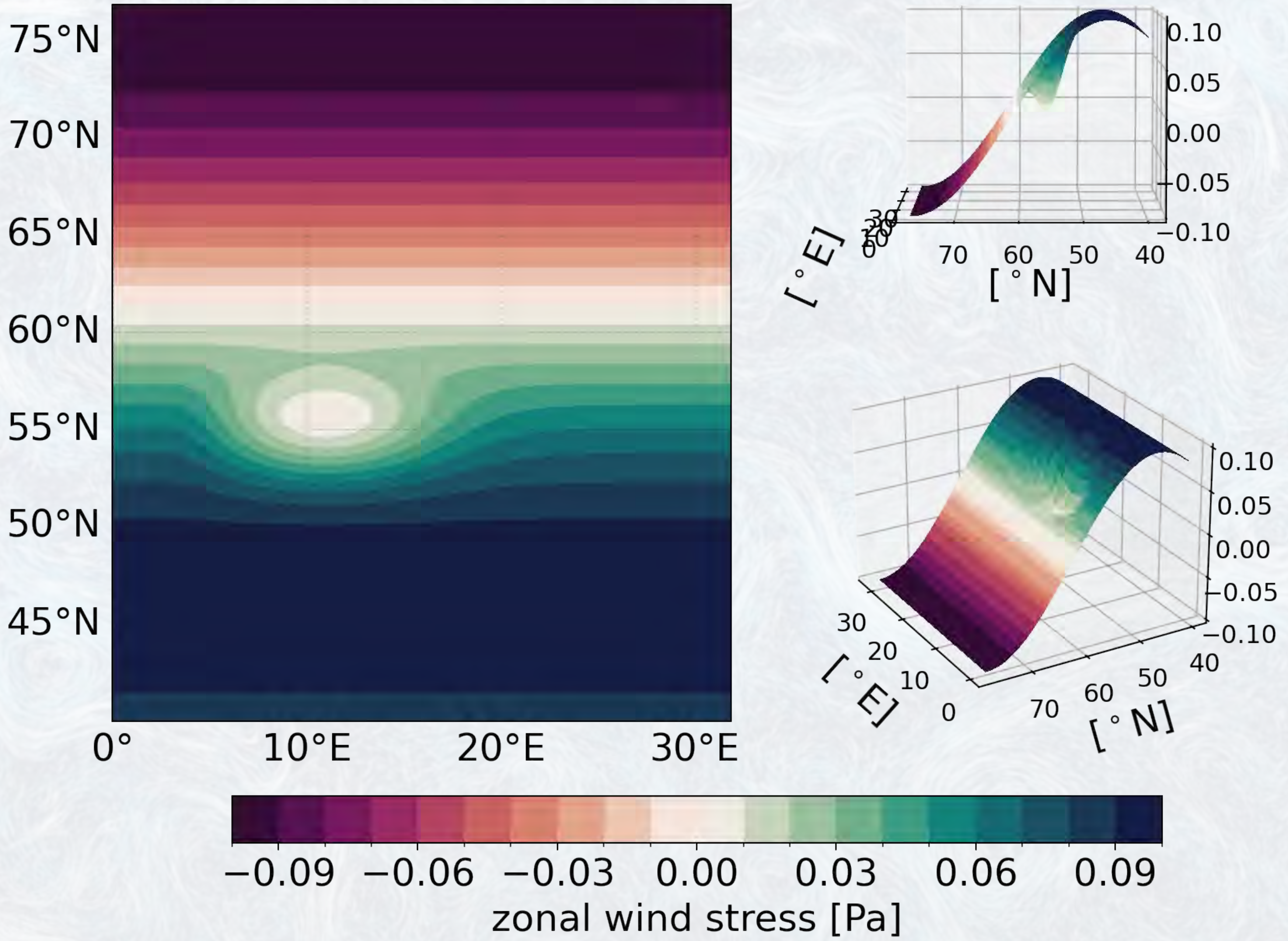
Synthetic forward model

Weibull distributed Beaufort state 1 winds

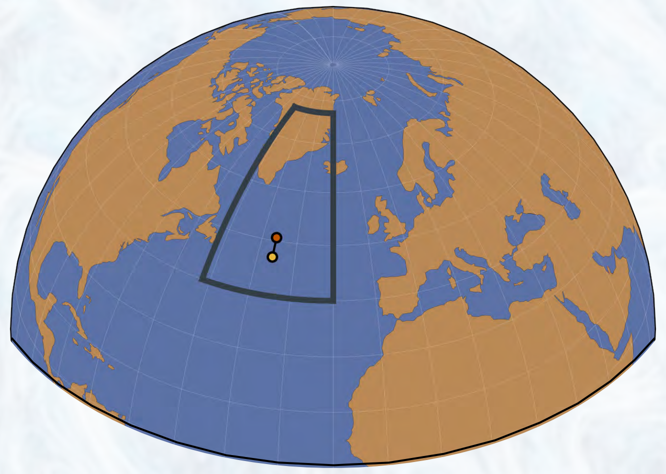


- Wind stress forcing modifies ocean state over two years after 80 years of spin up
- Observed travel times are hourly transmissions over 30 days

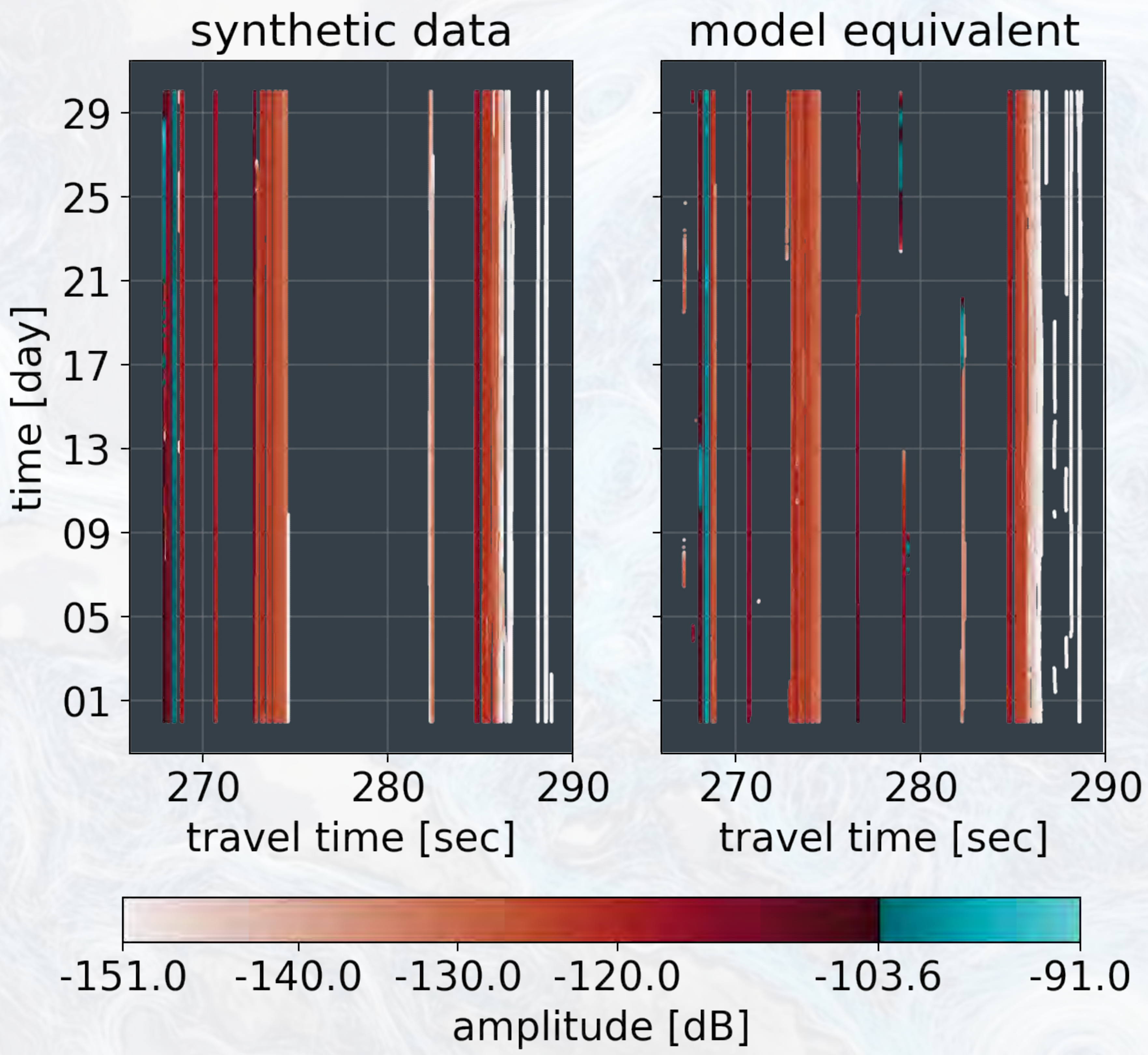
$$\tau_{\mathrm{obs}} \sim \boldsymbol{\tau} + \mathcal{N}(0, \sigma_\tau^2)$$

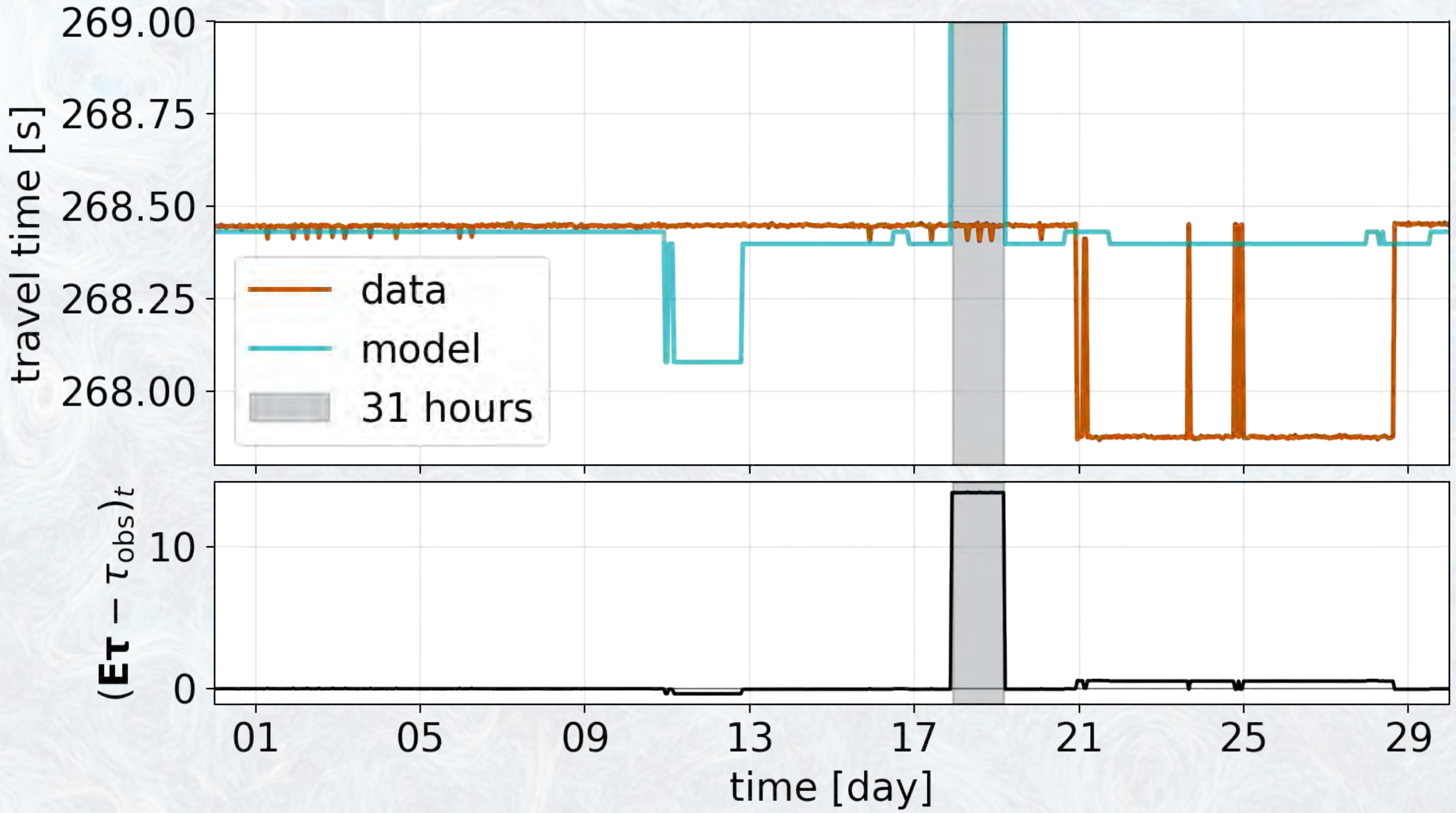


Model-data misfit of travel times



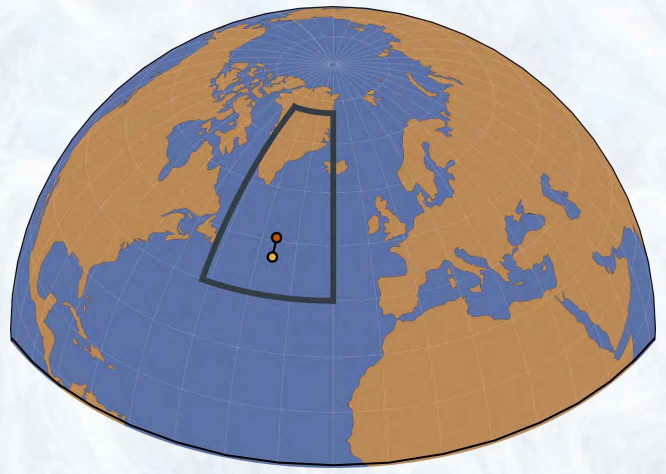
pto allows for unique ray paths between model and data

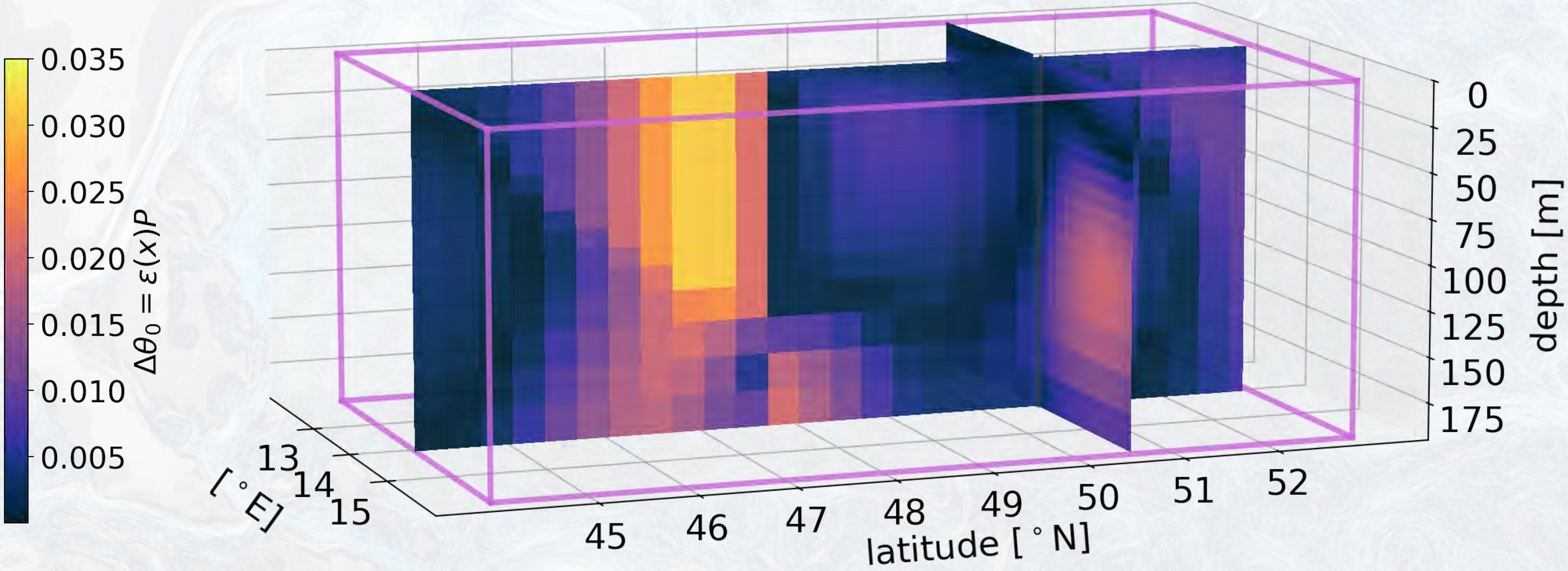




Temperature anomaly perturbation

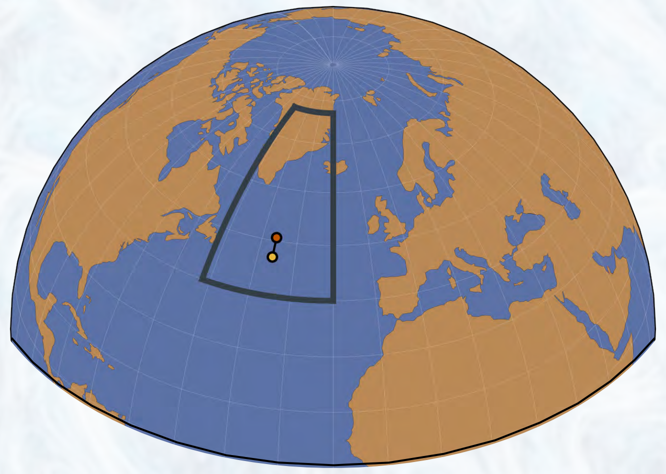
Apply nonuniform scaling to acoustic cost TLM





Acoustic cost function TLM

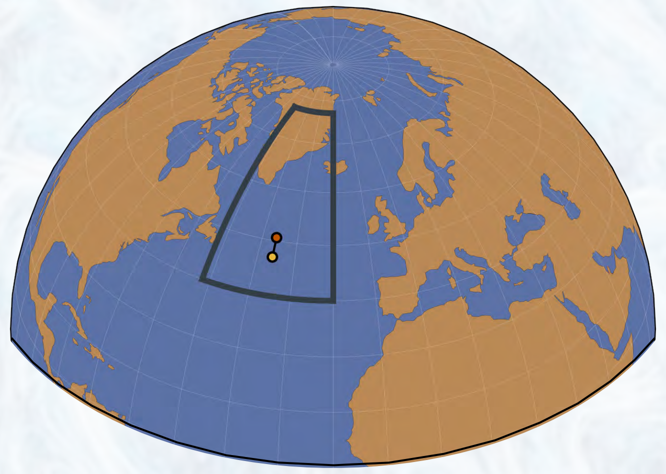
Directional derivative is recovered from uniform scaling



Reference state (forward model): $J = 5970.2823$		
Perturbation scale: ϵ	Directional derivative: $\delta J/\epsilon$	Cost increment: δJ
Nonuniform $(\Delta\boldsymbol{\theta}_0)$	—	-6.9033
10^0	-351.3624	-351.3624
10^{-1}	-351.3440	-35.1344
10^{-2}	-351.3226	-3.5132
10^{-3}	-351.3896	-0.3514
10^{-4}	-351.3285	-0.0351

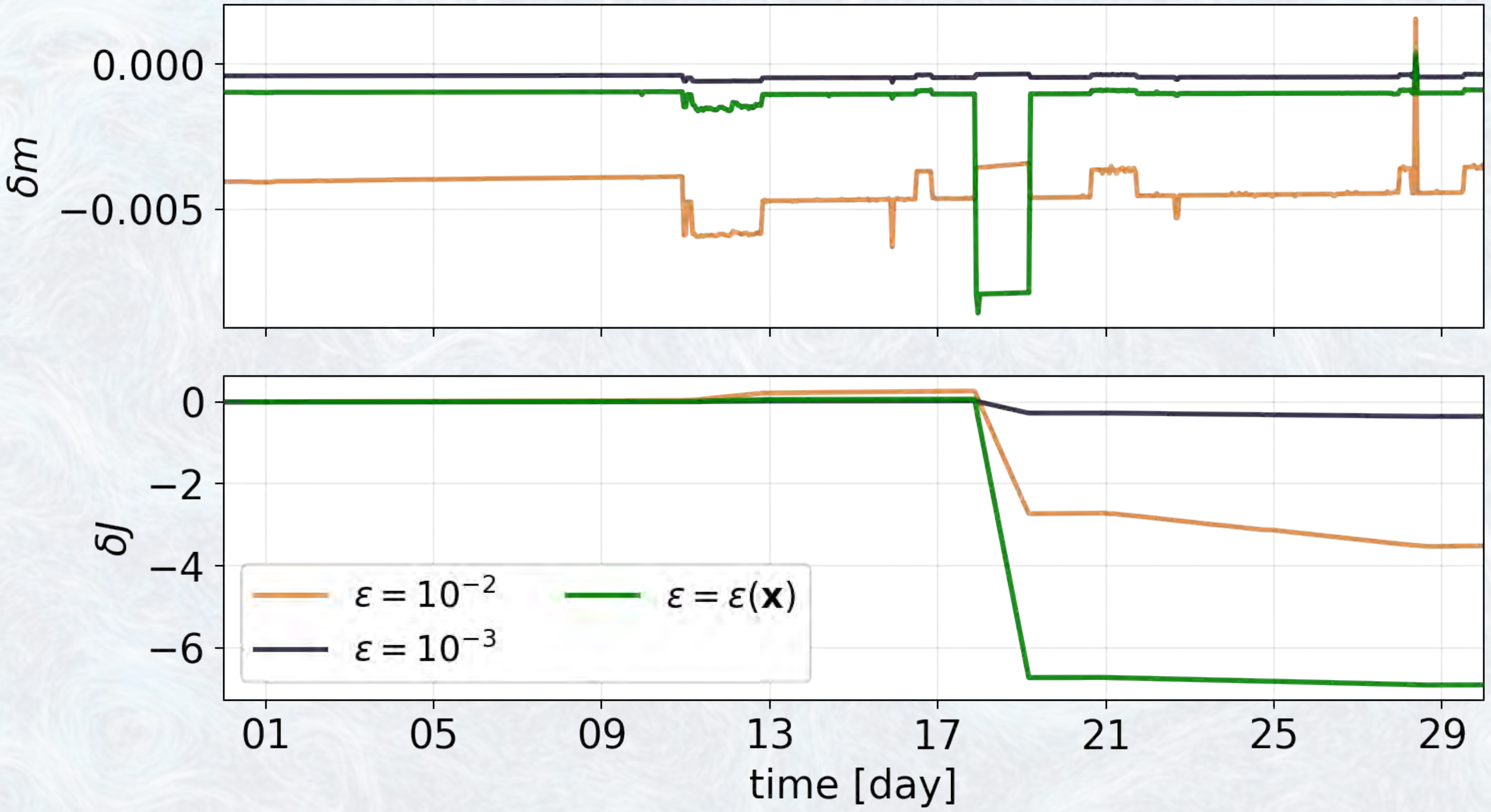
Acoustic cost and travel time increments

Distinct travel time response to spatial structure in temperature



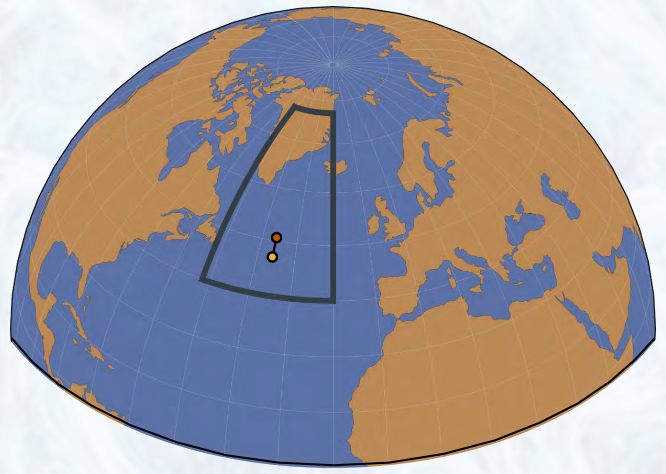
$$\delta J(t) = \delta m(t)^T \, \boldsymbol{E}^T \left(\boldsymbol{E\tau}(t) - \tau_{\text{obs}}(t) \right)$$

$$\delta J = \sum_{t=t_1}^{t_i} \delta J(t)$$

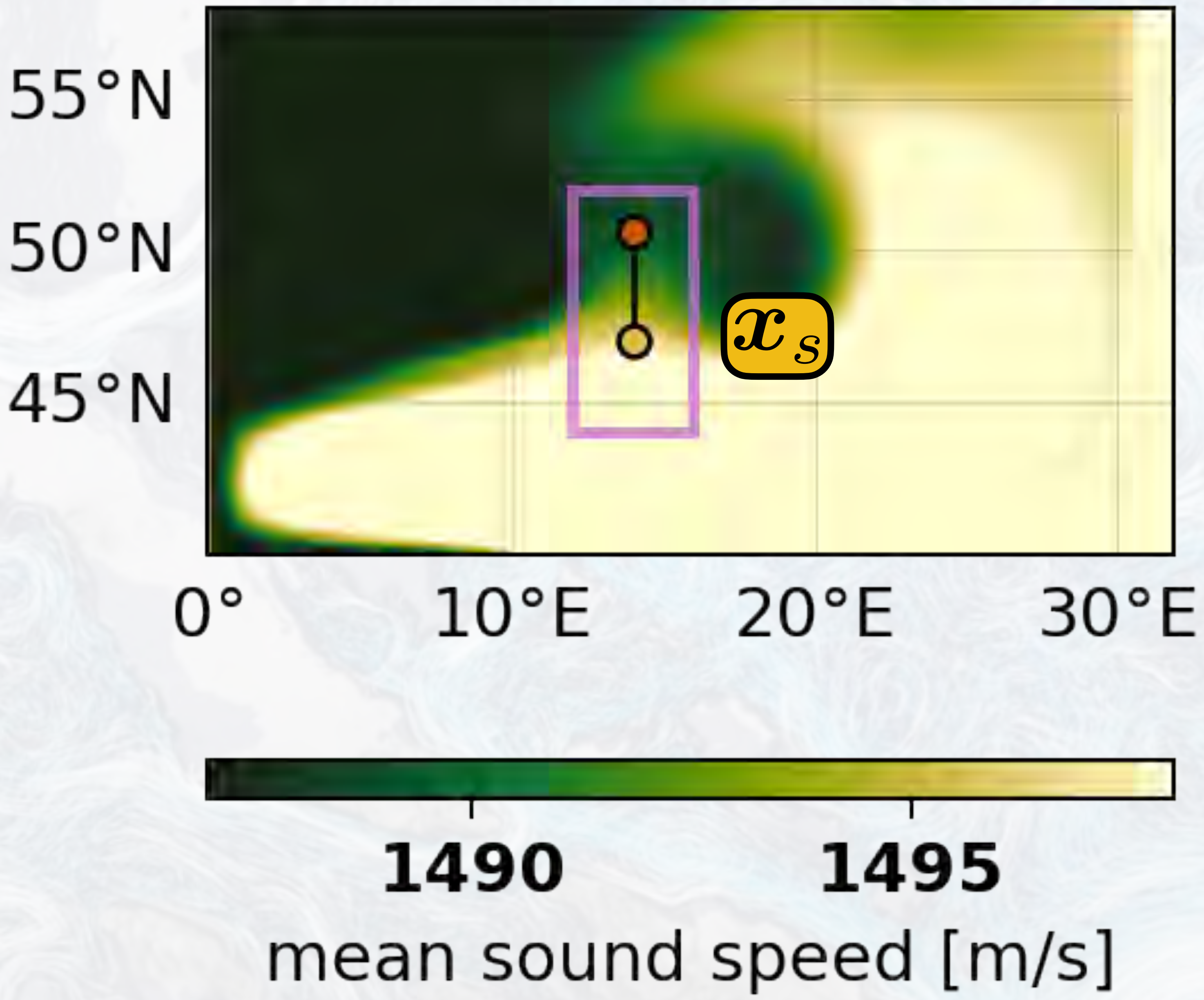


The acoustic cost increment

Pointwise assembly at a single vertical profile



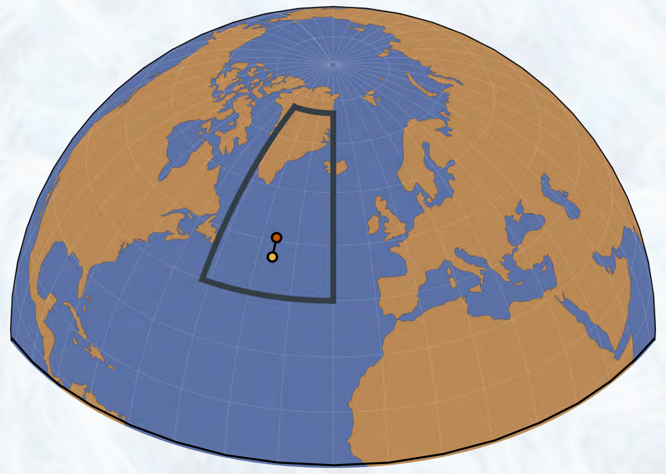
- Perturbing initial potential temperature at each depth contained in BoxSea at the sound source position

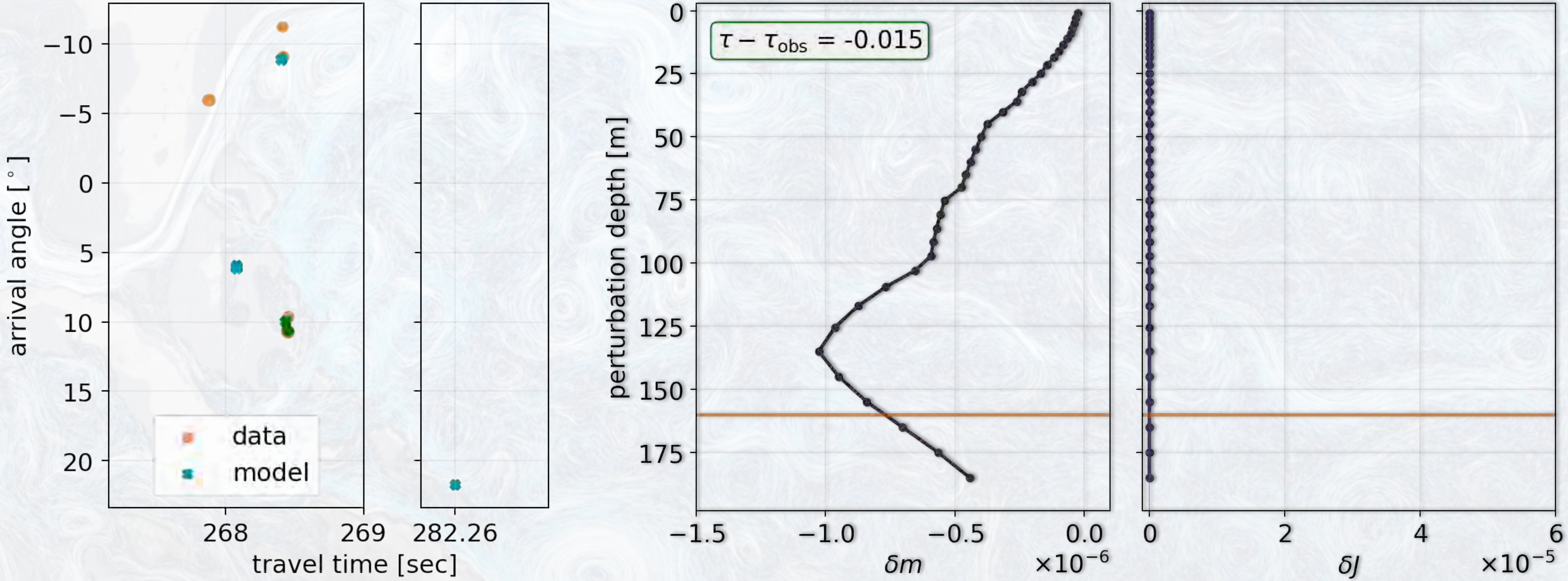


$$\Delta\boldsymbol{\theta}_0(z;\boldsymbol{x}_s) = \boldsymbol{P}(z), \quad \|\boldsymbol{P}\| = 1$$

The acoustic cost increment

Large misfit residuals accelerate descent towards observations





Summary

https://github.com/IvanaEscobar/**belli**

Highlights from the CSEM Ph.D. dissertation

- ◉ Coupled ocean-acoustic models
  - Added **belli**, an underwater acoustics package, to MITgcm
  - Opened MITgcm for hybrid compilation
- ◉ Differentiable underwater acoustics
  - Verified entire **belli** model with intermediate QoIs
  - Simplified AD NetCDF I/O routines for MPI runs
- ◉ Cost function of acoustic travel times
  - Discovered forward sensitivities due to changes in ocean temperature
  - Extended objective map to include acoustic observations

Efficient coupled framework that robustly differentiates through ocean-acoustic assimilation using underwater acoustic data

Thank you!

Questions?

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