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A post-hoc approach for including atmospheric error covariance in ocean state estimation

Dan Amrhein, **Shaunticlair Ruiz,** Ian Fenty, Ou Wang

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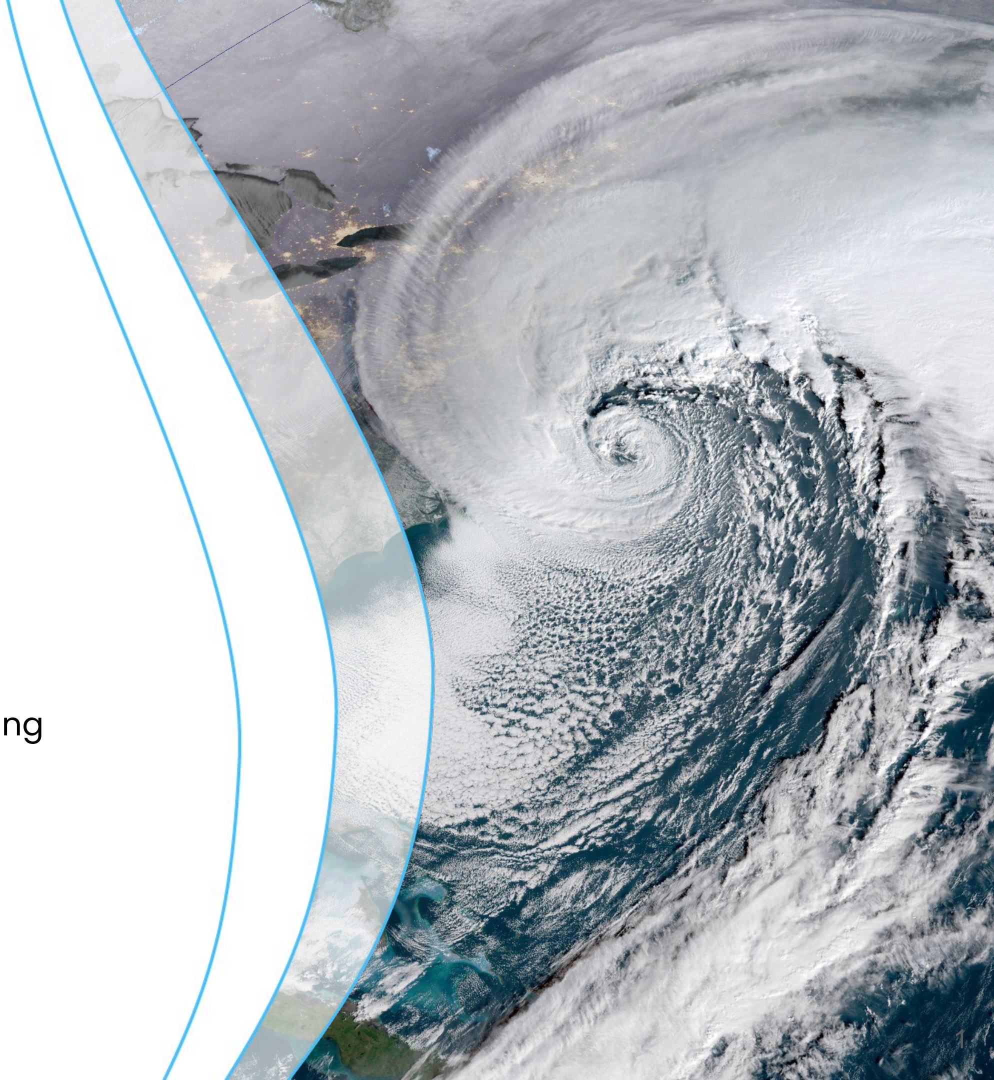
NSF National Center for Atmospheric Research



Entry point: *Atmospheric* updates can reflect where we have *ocean* data

A Global Glacial Ocean State Estimate Constrained by Upper-Ocean Temperature Proxies

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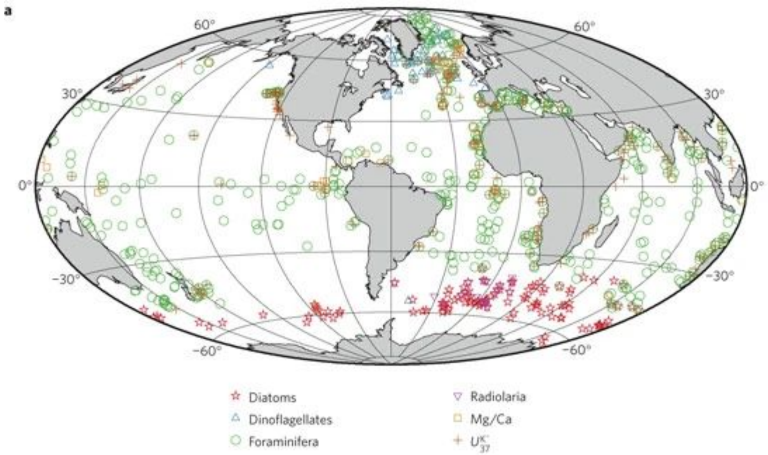
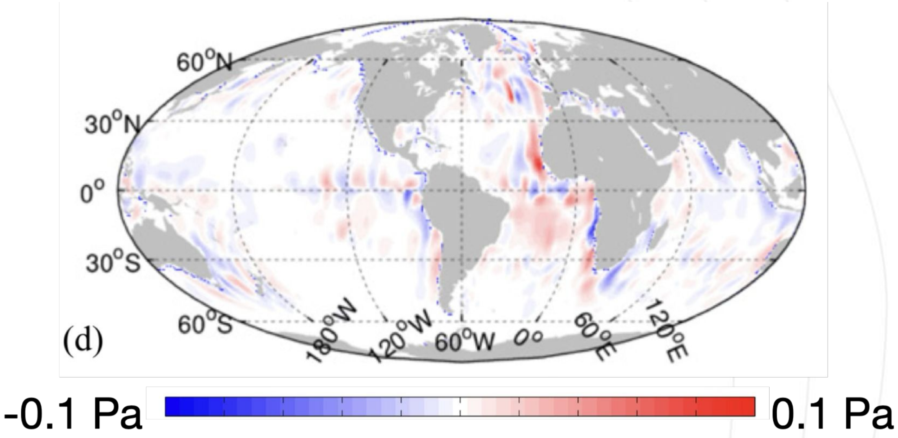


Data locations



Control updates ($\boldsymbol{\tau}_v$)
Does the atmosphere do this??

Some grief about loss (function)

This is what ECCO minimizes *(Wunsch 2006)*:

$$J = (\mathbf{Ex} - \mathbf{y})^T \mathbf{R}_{nn}^{-1} (\mathbf{Ex} - \mathbf{y}) + \mathbf{u}^T \mathbf{Q}^{-1} \mathbf{u} - 2\mu^T (\mathbf{Ax} - \mathbf{b} - \mathbf{\Gamma u})$$

Fit the data!
(within uncertainty)

…OK but don't go ***too*** *crazy with changing controls*

Obey model physics!
(and do it exactly)

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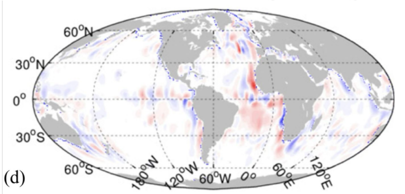
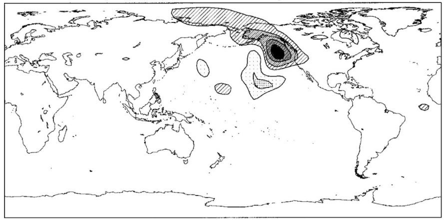
ECCO uses a diagonal **Q** (+ `smooth` package)
Who cares?
1) It can be unphysical / icky
2) Atmospheric errors have **structure** and we are throwing away information!





Houtekamer and Mitchell (1998)

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Hypothesis: Using **Q** from atmospheric unc'ty gives a better state estimate.

(Side note: It should be possible to implement in *J* without big matrices…)

Here: building a more sophisticated `smooth` approach

This work: A statistically informed `smooth`

Procedure: Require that the projection **u** onto sensitivities **s** gives a desired loss function change δ

Add a soft (Mahalanobis) constraint that **u** agree with statistics **C**

related:

An Assessment of Uncertainty in the ECCO Global Ocean-Sea Ice State Estimate Due To Atmospheric Forcing Uncertainty

Yanzhou Wei[1,2,3], Helen Pillar[1], Patrick Heimbach[1,4], An T. Nguyen[1], Gaël Forget[5], Ou Wang[6], Ichiro Fukumori[6], Ian Fenty[6], and Martin Losch[7]

This work: A statistically informed `smooth`

Procedure: Require that the projection **u** onto sensitivities **s** gives a desired loss function change δ

Add a soft (Mahalanobis) constraint that **u** agree with statistics **C**

$$\mathcal{L}(\mathbf{u}) = 2\boldsymbol{\lambda}\left(\mathbf{u}^\top \mathbf{s} - \boldsymbol{\delta}\right) + \mathbf{u}^\top \mathbf{C}^{-1}\mathbf{u}$$

This work: A statistically informed `smooth`

Procedure: Require that the projection **u** onto sensitivities **s** gives a desired loss function change δ

Add a soft (Mahalanobis) constraint that **u** agree with statistics **C**

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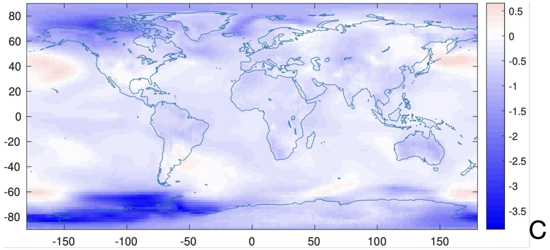
Minimizing gives

$$\mathbf{u} = \boldsymbol{\delta}\left(\frac{\mathbf{Cs}}{\mathbf{s}^\top \mathbf{Cs}}\right)$$



Promising at LGM!

Hypothesis for Shaunticlair's internship:

Statistical smoothing improves state estimation in a simplified model.

Shaunticlair Ruiz

Toy Model: Advection-Diffusion-Forcing Model

- **Tracer**: quantity of interest, heat
- **Advection**: Ocean currents carry heat
- **Diffusion**: Heat moves from hot to cold
- **Forcing**: Atmosphere adds heat flux
- Equations discretely approximated to **first order**

$$\frac{\partial c}{\partial t} = \overbrace{D\nabla^2 c}^{\text{Diffusion}} - \overbrace{\nu\nabla c}^{\text{Advection}} + \overbrace{F(f-c)}^{\text{Forcing}}$$

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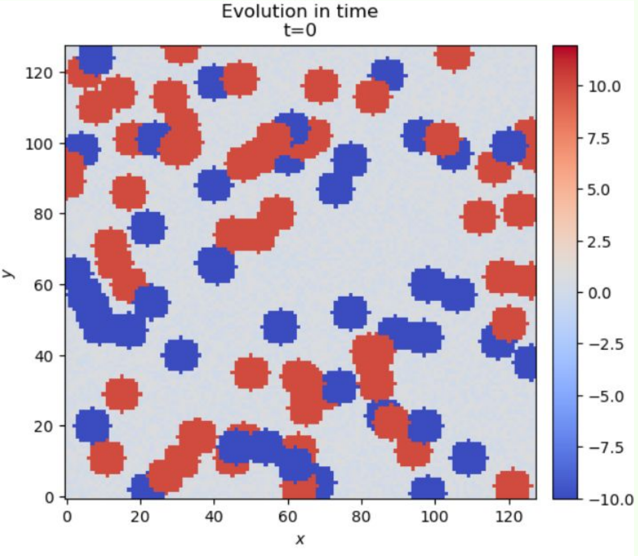
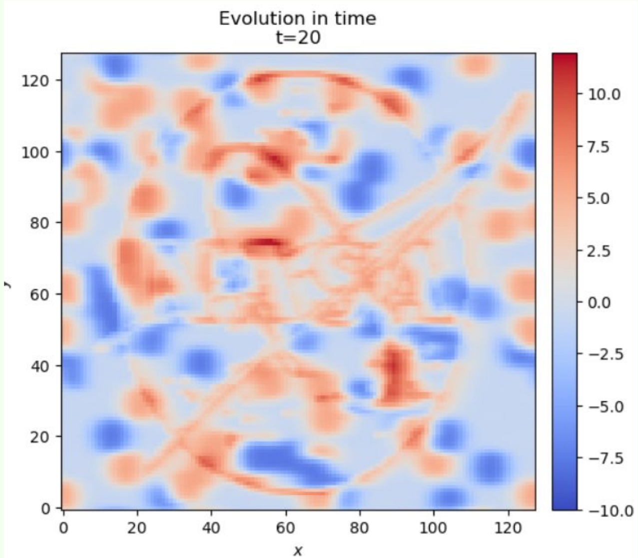
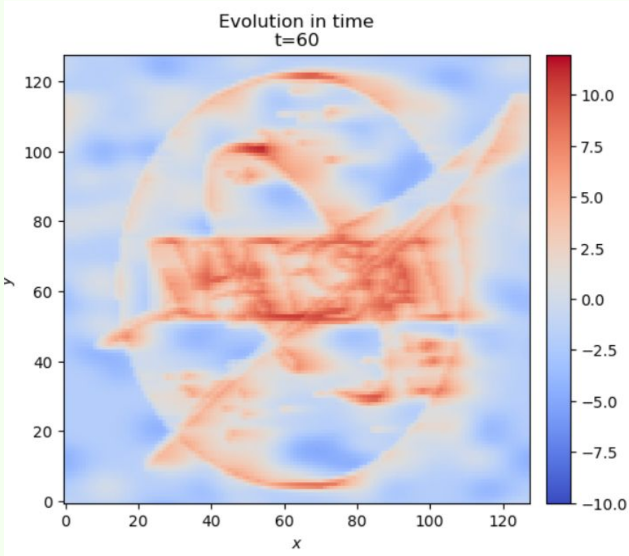
Toy Model demo (diffusion only)







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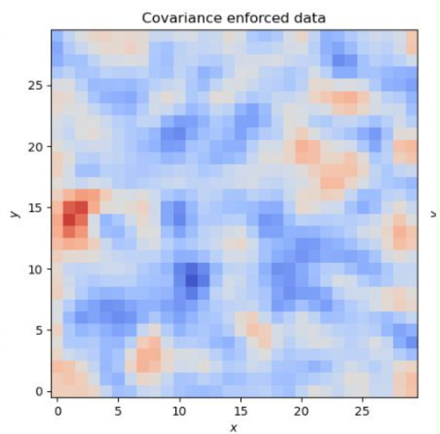
Forcing covariance Matrix: Gaussian decay

$$C_{a,b} = \exp\left(\frac{-\|a-b\|^2}{s}\right)$$



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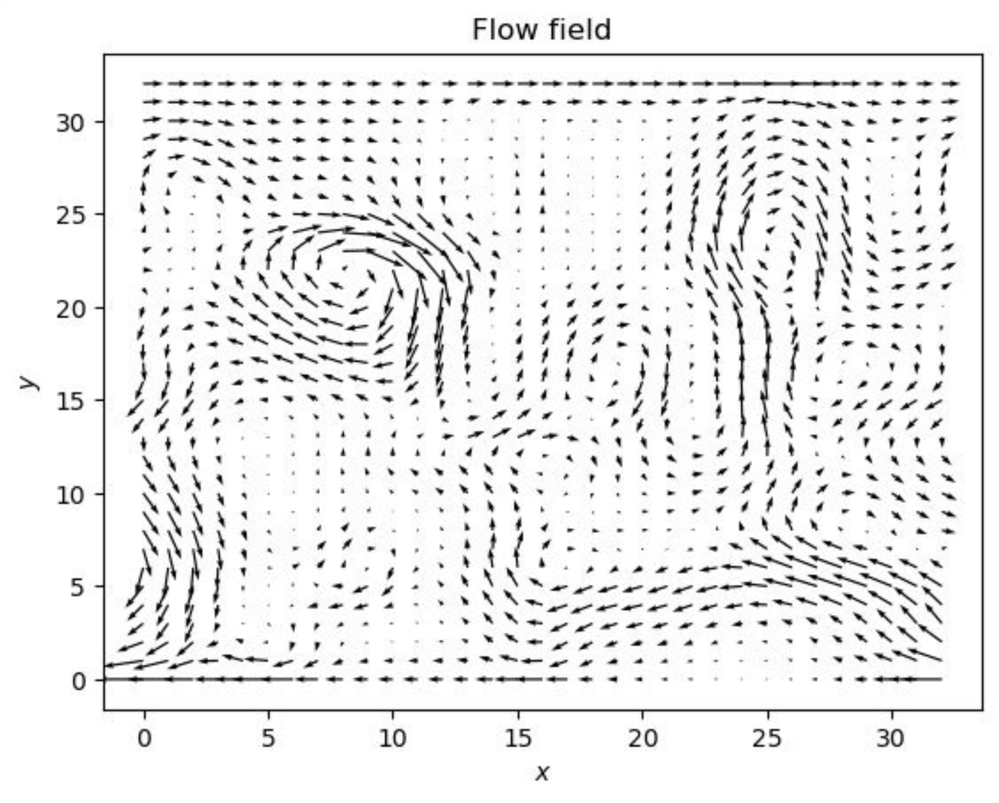
Experimental design



- Random initial ocean state
- True and first-guess atmospheres both have covariance C
- Randomly observe “true” ocean state with some noise
- Use dJ/d**f** for gradient descent
  - Computed via the **adjoint method**

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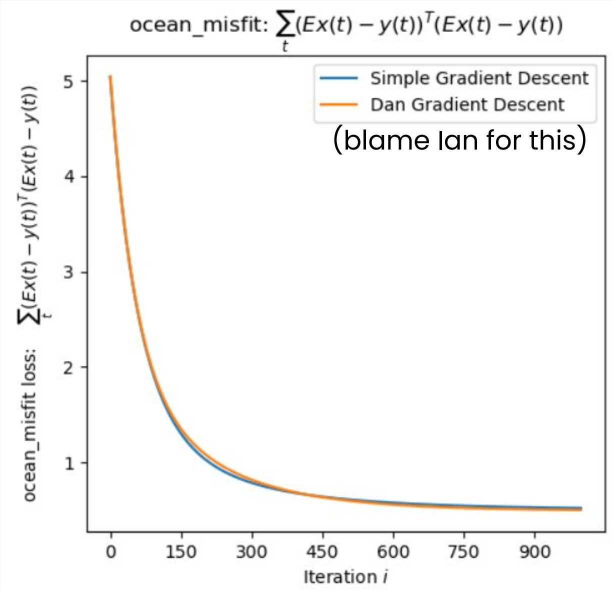
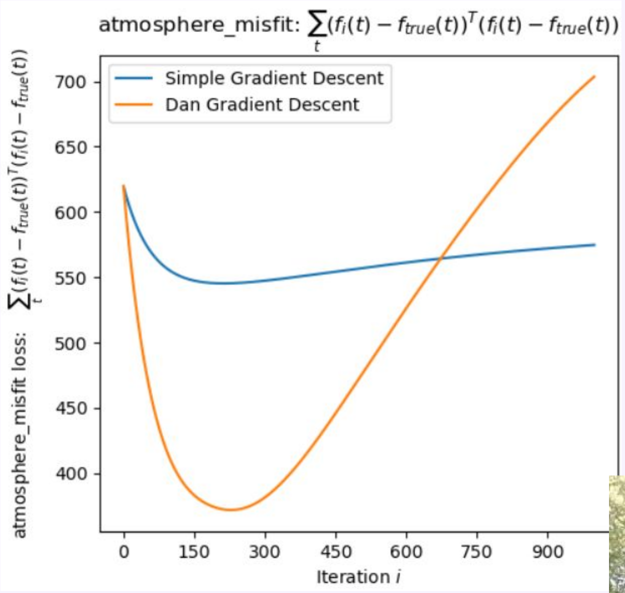
Statistical smoothing creates a better fit for gradient descent

- Left: J, the ocean misfit
- Right: atmosphere misfit





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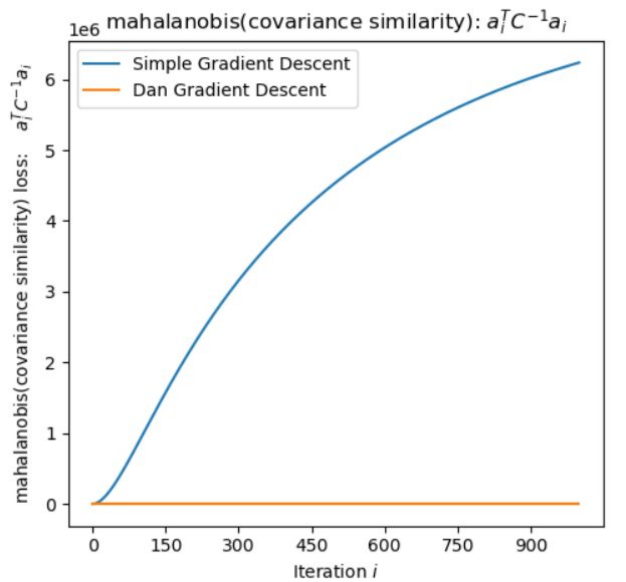

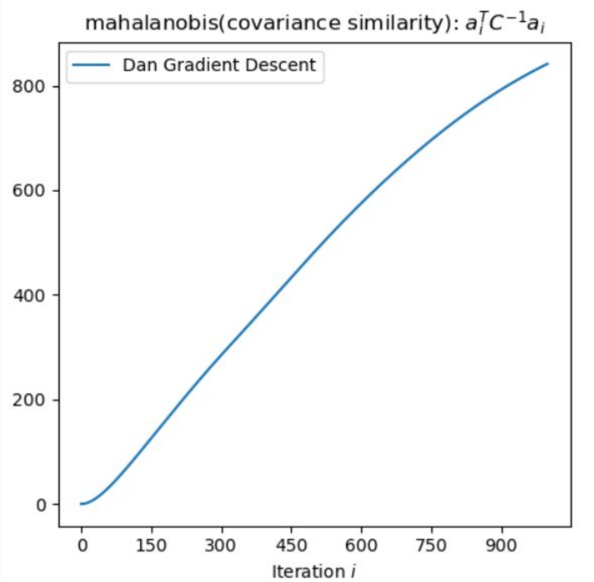

- **Mahalanobis distance** measures similarity of inferred forcing covariance, to true covariance C

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Conclusions and next steps

Bottom line: Smoothing to make control adjustments obey covariance relationships is effective in a simple model.
To do: Scaling changes may be needed for multiple iterations
Next step: Try in a real state estimate! Use spread from atm. reanalysis
Caveat: Control adjustments also correct ocean biases. Could subtract these out

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Workshop: Regional MOM6 in CESM. DA! CICE! WW3! Travel funds available.

CrocoLake: DA pipeline for ocean obs