OCEAN & SEA ICE BUDGETS IN ECCO
Part 1: Overview & Applications

Helen Pillar (Helen.Pillar@utexas.edu)

University of Texas at Austin, Austin, TX

INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES
ODEN

Estimating the Circulation and Climate of the Ocean
ECCO
National Aeronautics and Space Administration

With thanks to:

Chris Piecuch

An Nguyen

What Drives the Steady and Time-Varying Ocean & Sea Ice States?

Seek explanations that are robust, quantitative & mechanistic

What are budgets are why are they useful?

Property evolution & conservation:

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla \mathbf{v} - f\hat{\mathbf{k}} \times \mathbf{v}_h - \frac{1}{\rho_c} \nabla_h p + \frac{1}{\rho_c} \mathcal{F}_v$$

$$\frac{\partial \theta}{\partial t} = -\mathbf{v} \cdot \nabla \theta + \frac{1}{\rho_c C_p} \mathcal{H}_\theta$$

$$\frac{\partial S}{\partial t} = -\mathbf{v} \cdot \nabla S + \frac{1}{\rho_c} \mathcal{Q}_S$$

Obtaining a CLOSED budget:

- tendency = sources + sinks + redistributions
- provides meaningful insights into drivers of change

How to assess budgets?

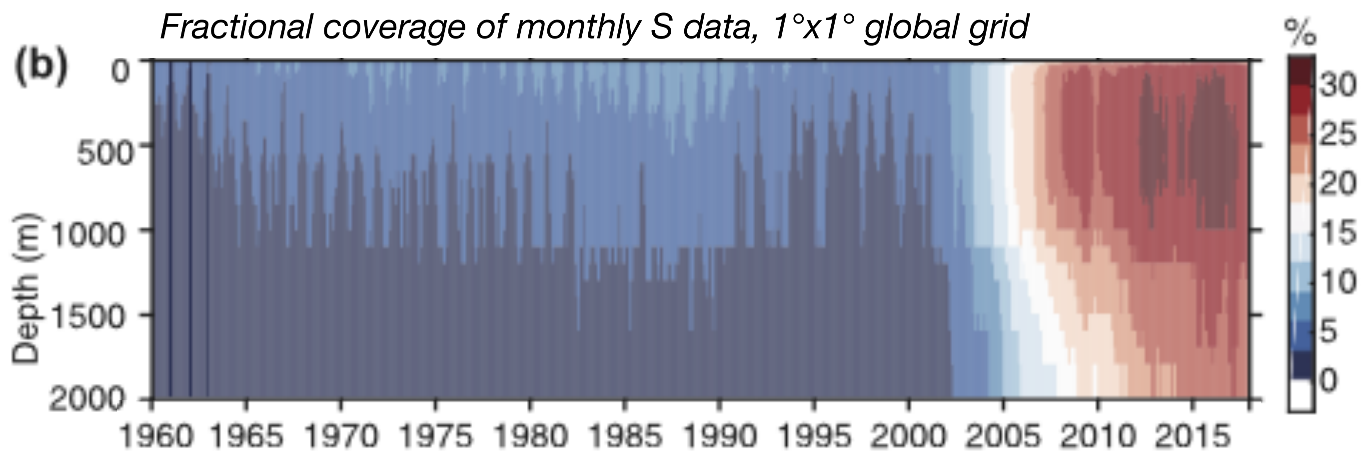
- Observations?
- Data products?
- Model-data syntheses?
  - Ocean reanalyses?
  - Ocean state estimates?

Observations

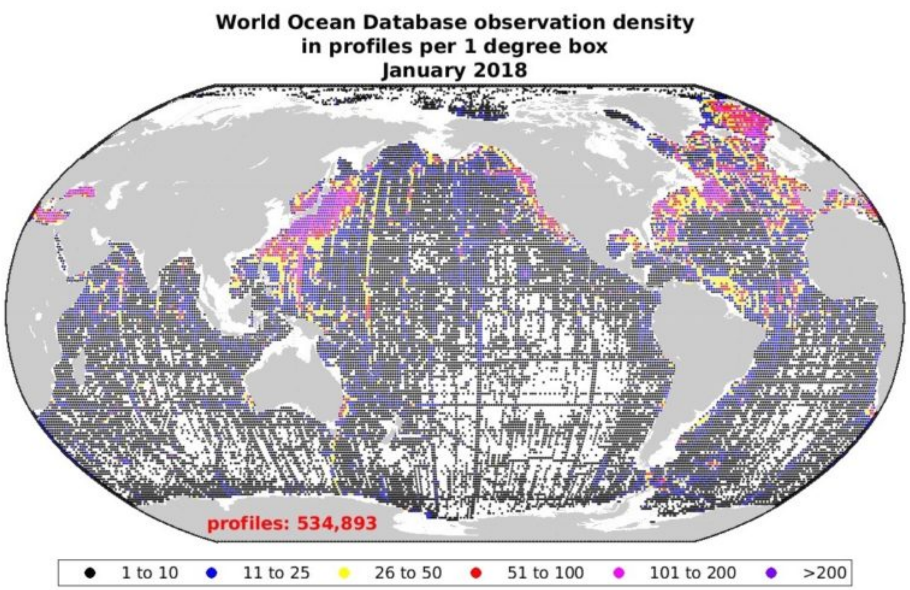
&

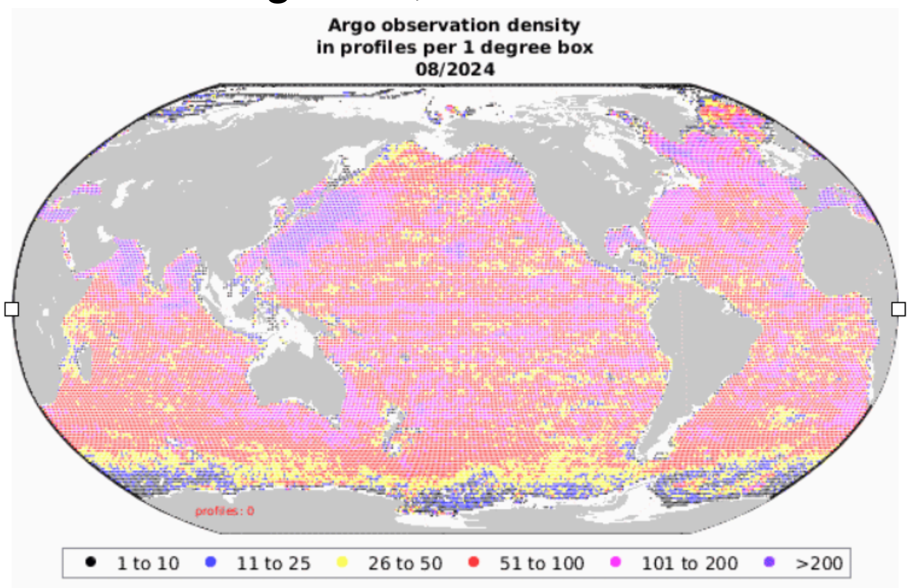
Data Products

Assessing Budgets: Challenges using observations



Cheng et al., 2016

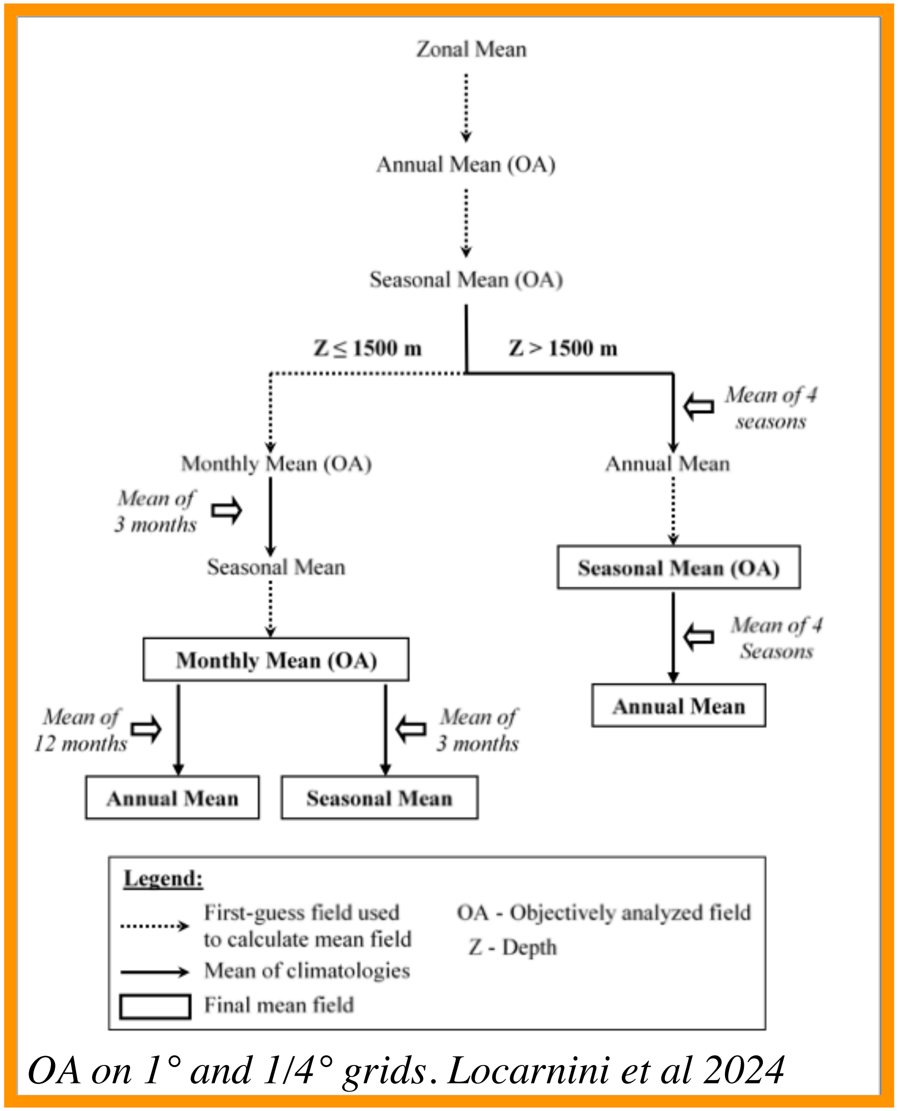




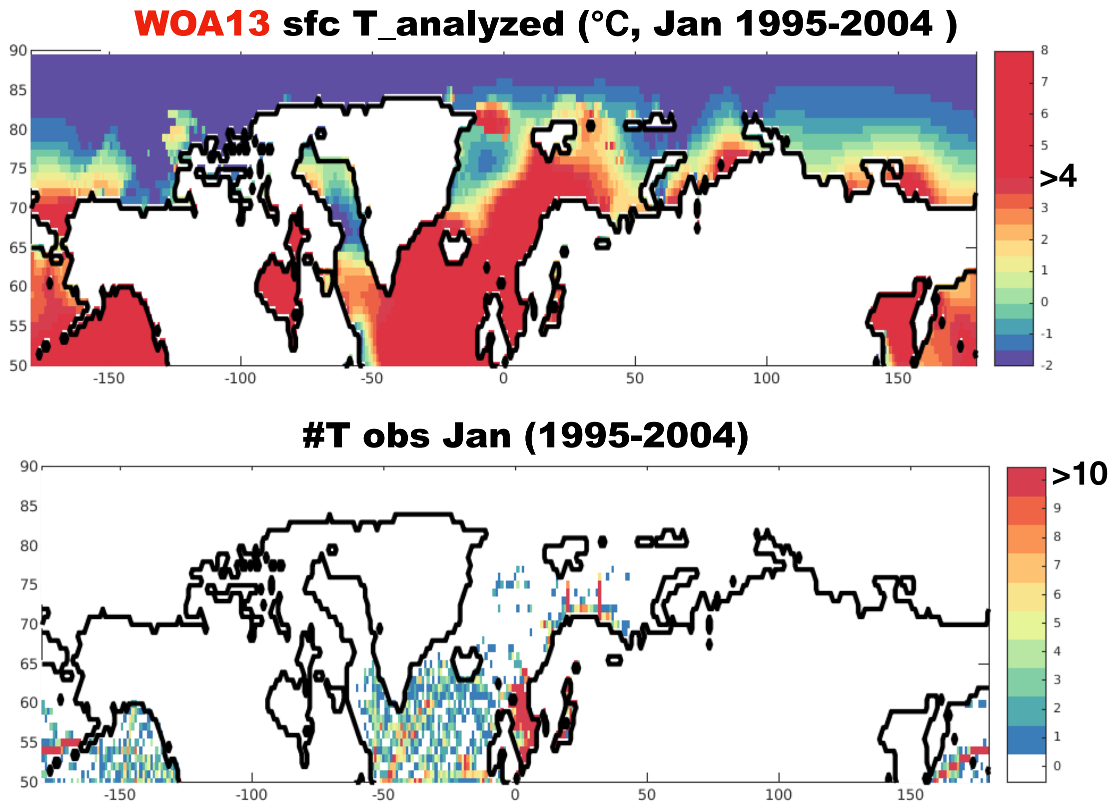
(https://www2.whoi.edu/site/argo), Johnson et al., 2022)

- incomplete, sampled heterogeneously in space & time

Assessing Budgets: Challenges using data products



OA on 1° and 1/4° grids. Locarnini et al 2024



- Statistical gap-filling (e.g., objective analysis) is unreliable in data poor regions; Arctic analysis biased warm

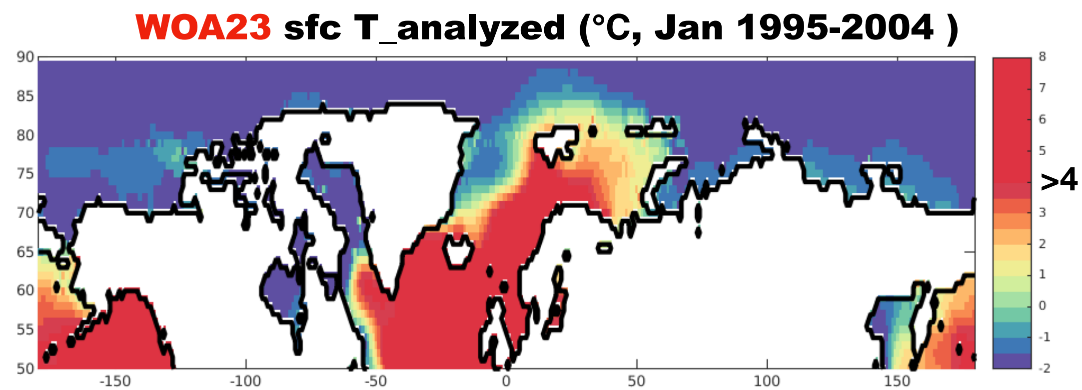
Assessing Budgets: Challenges using data products

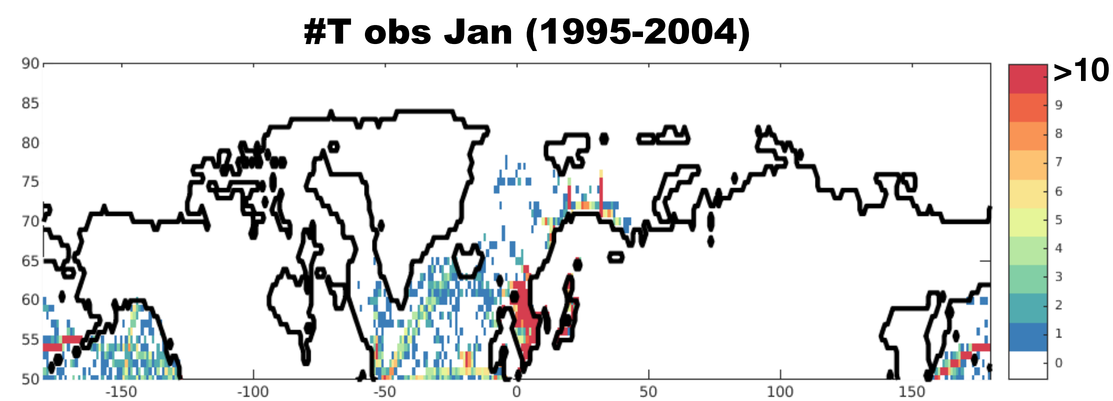
"We are still hampered in a number of ways by the lack of data…

Data may exist in one area for one season, thus precluding any representative annual analysis…

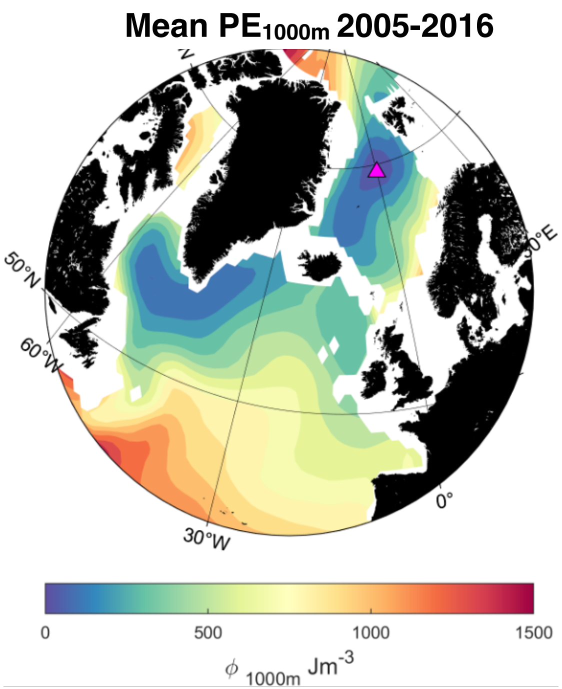
We provide the number of observation fields so data reliability can be assessed by the user"

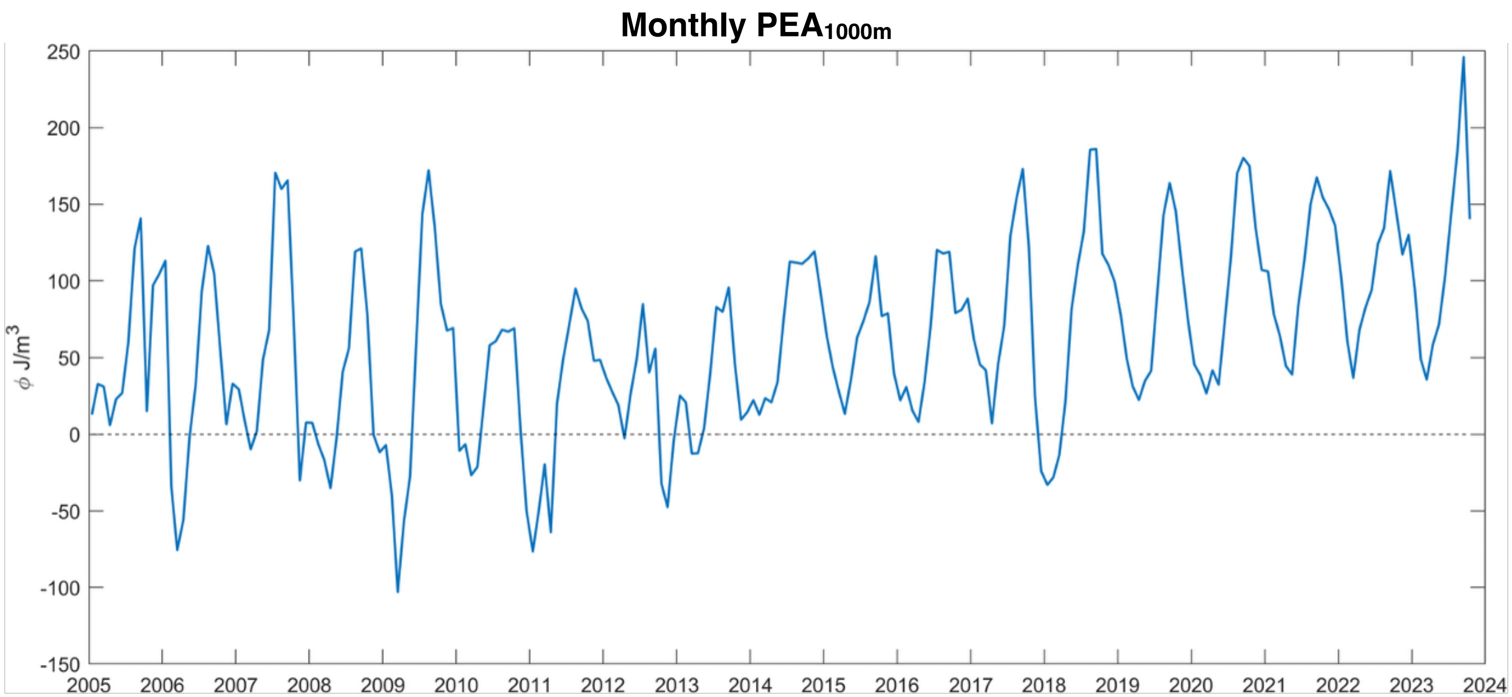
Locarnini et al., 2024





Assessing Budgets: Challenges using data products

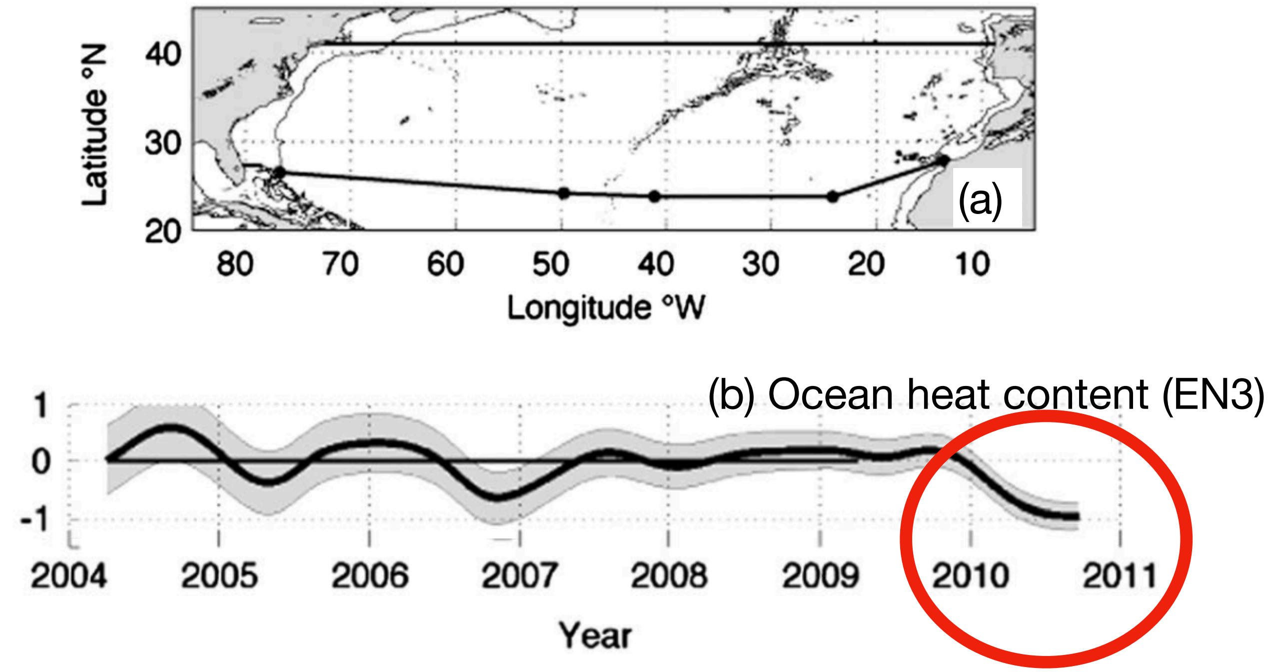




- ...and can introduce unphysical artifacts e.g., widespread sustained static instability in EN4 gridded product

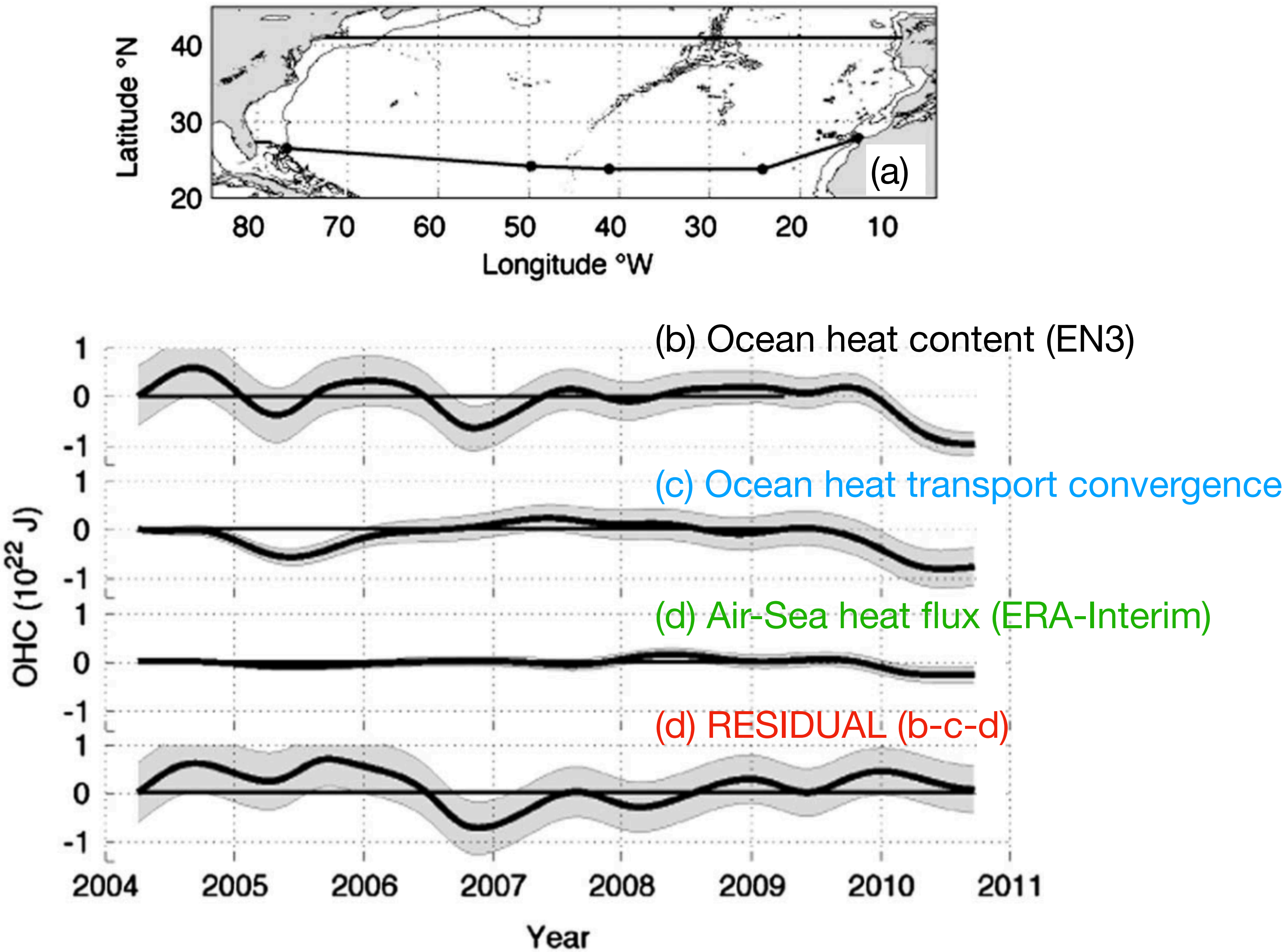
Courtesy of Ben Lincoln, Bangor University

Assessing Budgets: Challenges using data products



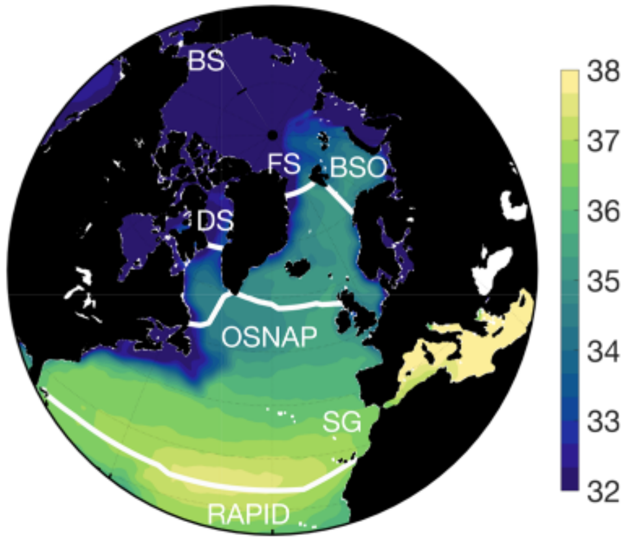
Cunningham et al., 2013: Exploring drivers of the subtropical Atlantic cold anomaly

Assessing Budgets: Challenges using data products



Cunningham et al., 2013: Exploring drivers of the subtropical Atlantic cold anomaly

Assessing Budgets: Challenges using data products

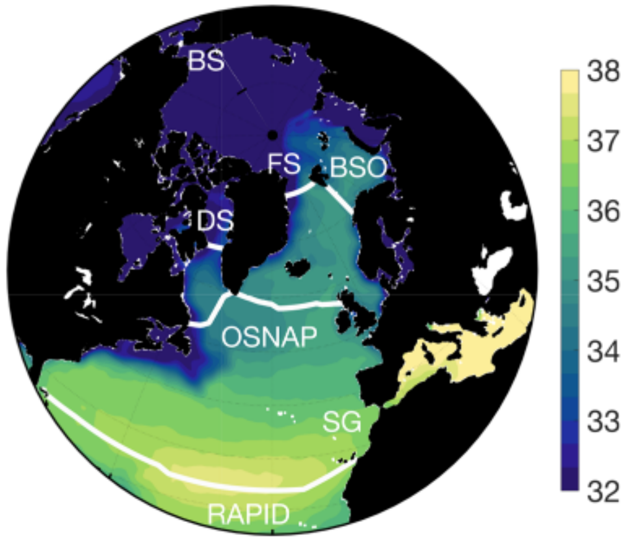


North Atlantic freshwater budget (Sv)

	OSNAP	RAPID	Strait of Gibraltar	FW_{adv}	FW_{OSC}
8/2014–5/2018	−0.36 ± 0.05	0.42 ± 0.03	0.03 ± 0.01	0.09 ± 0.06	−0.15 ± 0.09
4/2004–8/2018	−0.36 ± 0.07	0.43 ± 0.02		0.10 ± 0.07	−0.05 ± 0.02
	a				

Li et al., 2021: Quantifying North Atlantic air-sea freshwater exchange

Assessing Budgets: Challenges using data products



North Atlantic freshwater budget (Sv)

	OSNAP	RAPID	Strait of Gibraltar	FW_{adv}	FW_{OSC}	FW_{sfc}	Independent reanalyses		
						Observation	NCEP CFSv2	JRA55	ERA5
8/2014–5/2018	−0.36 ± 0.05	0.42 ± 0.03	0.03 ± 0.01	0.09 ± 0.06	−0.15 ± 0.09	−0.06 ± 0.11	−0.11 ± 0.01	−0.19 ± 0.01	−0.14 ± 0.01
4/2004–8/2018	−0.36 ± 0.07 [a]	0.43 ± 0.02		0.10 ± 0.07	−0.05 ± 0.02	0.05 ± 0.07	−0.11 ±0.01[b]	−0.19 ±0.005	−0.15 ± 0.004
	+	+	=	+	=				

Li et al., 2021: Quantifying North Atlantic air-sea freshwater exchange

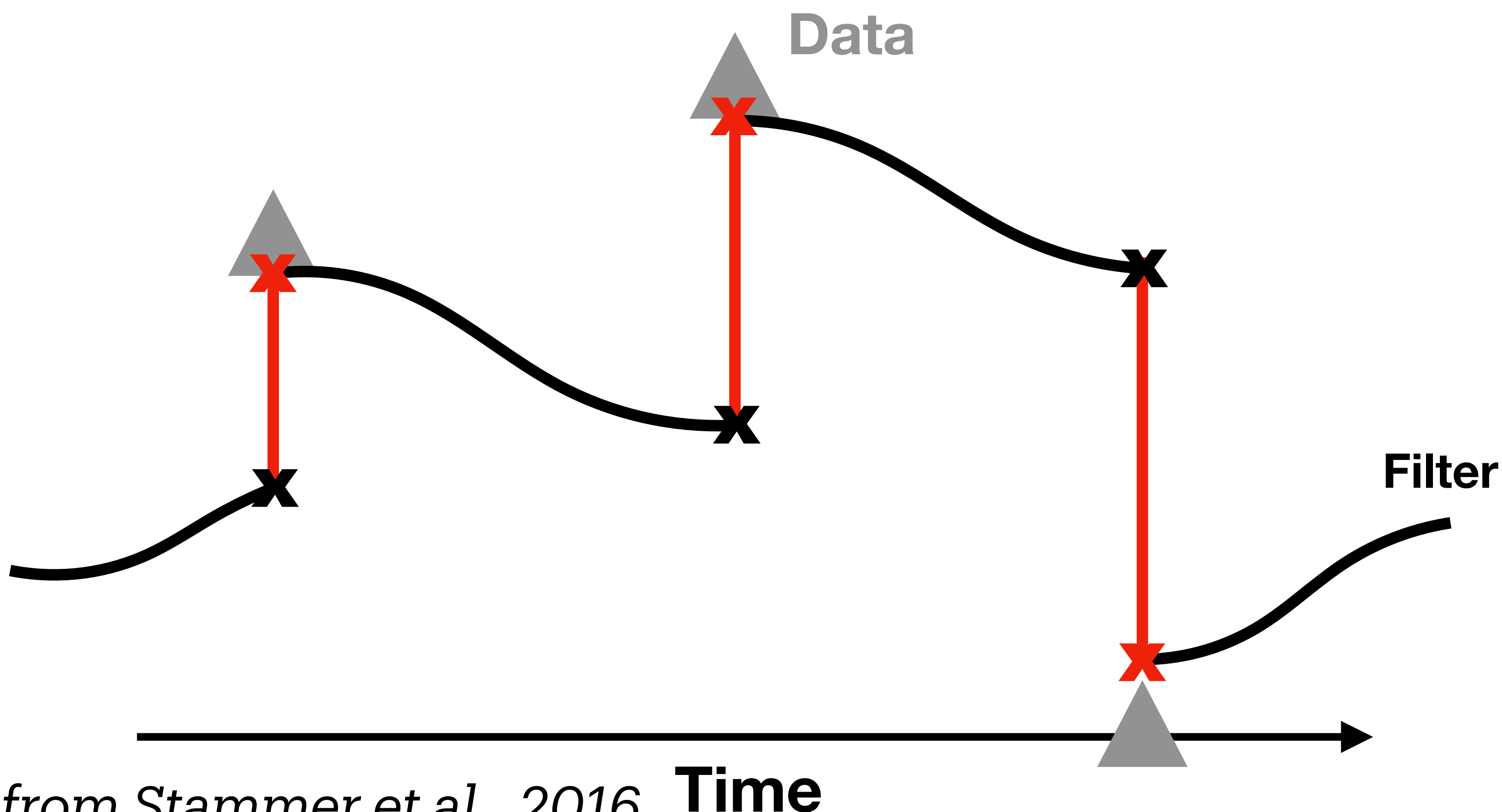
Reanalyses

&

State Estimates

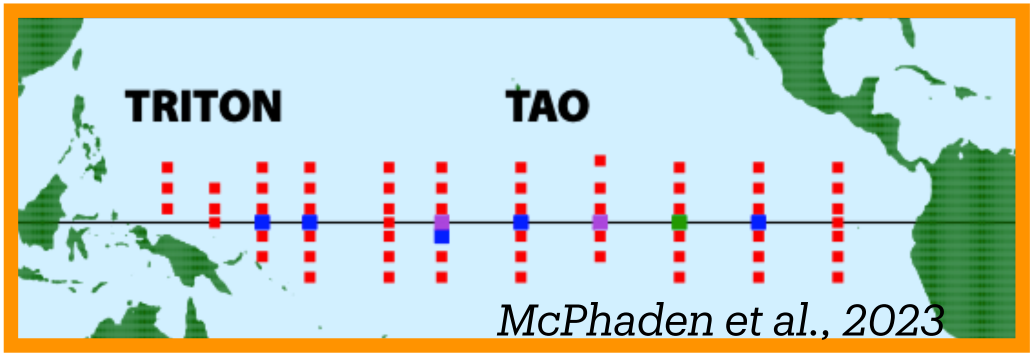
Assessing Budgets: Challenges using reanalyses

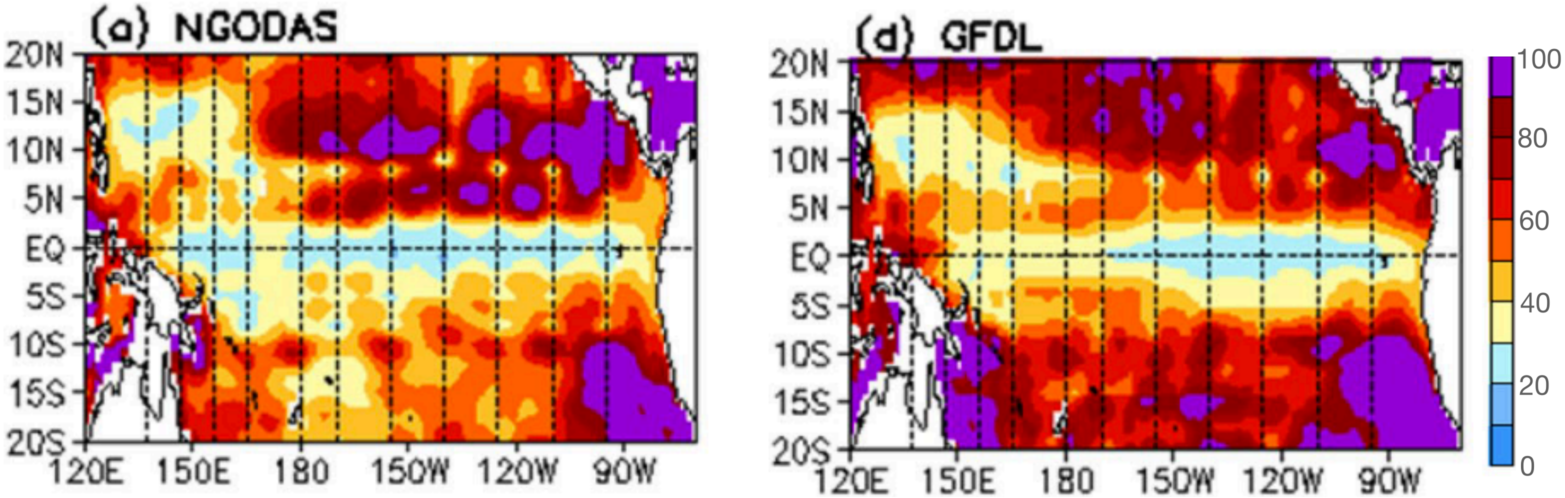
- Target = optimal forecast via IC adjustment
- Dynamical consistency/property conservation *NOT* enforced
- Analysis increments introduce spurious sources/sinks



Adapted from Stammer et al., 2016

Assessing Budgets: Challenges using reanalyses

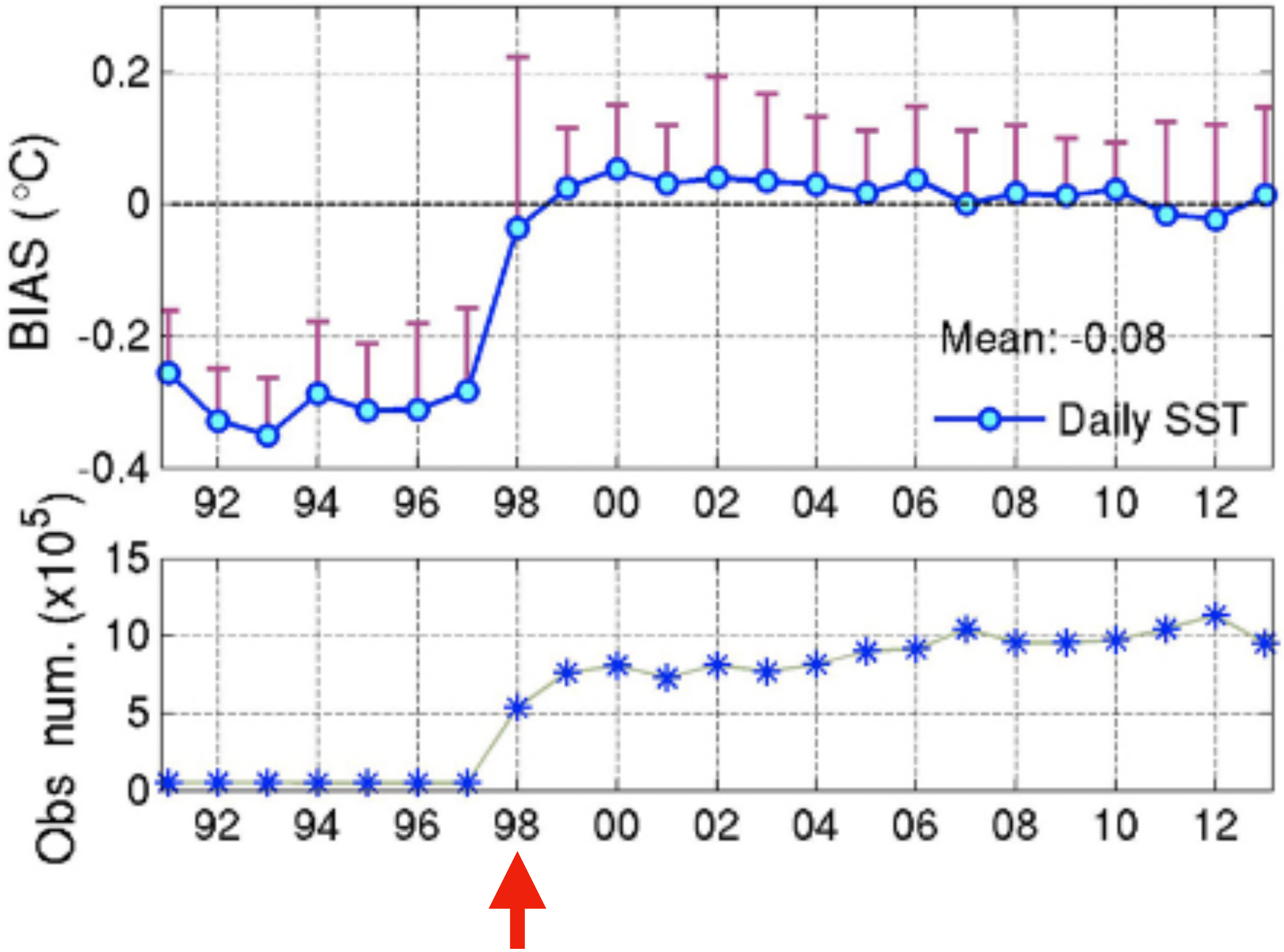




T'$_{300m}$ rms error (%) wrt reanalysis ensemble mean

Xue et al 2017: Strong overfitting to local constraints and generation of spurious variability between moorings

Assessing Budgets: Challenges using reanalyses

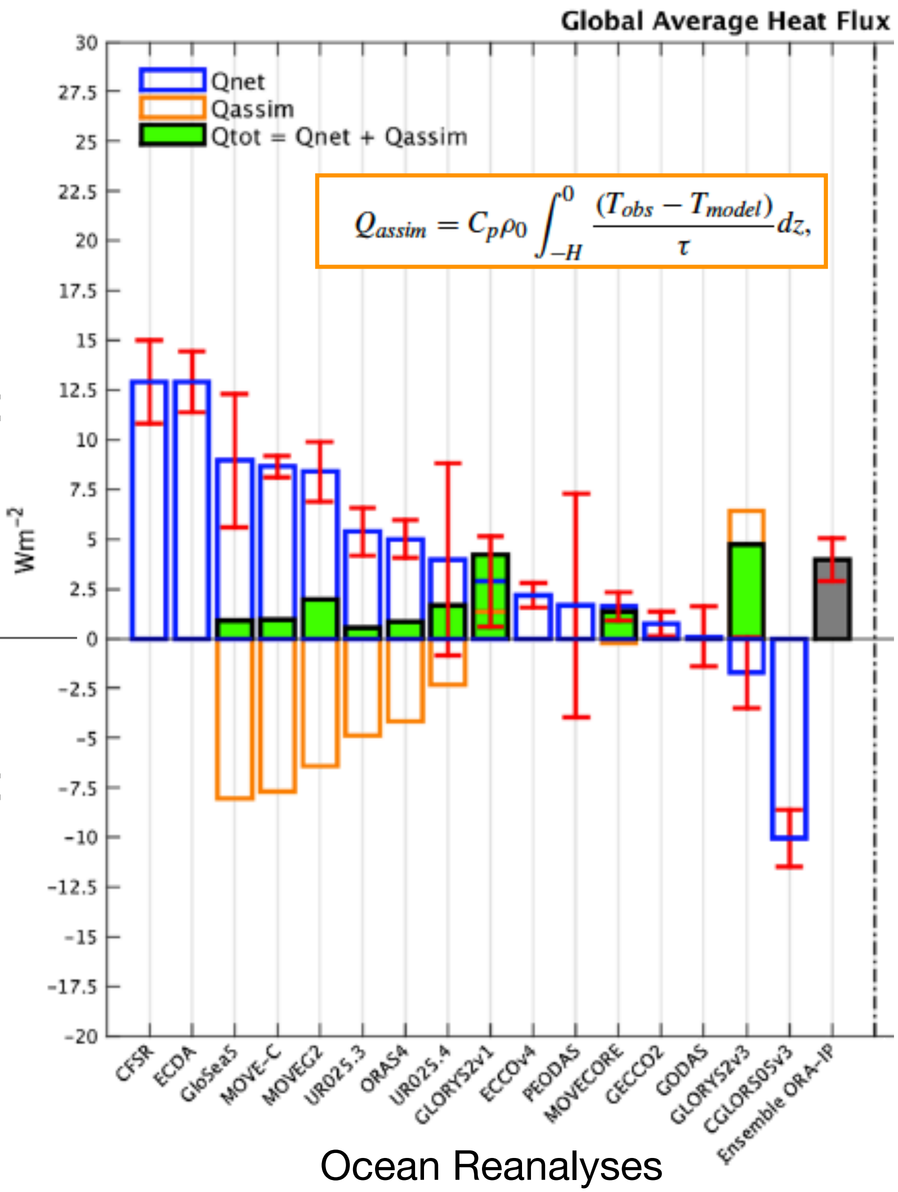


Xie et al., 2017: Discontinuities in TOPAZ4 Arctic SST due to observing system changes

Assessing Budgets: Challenges using reanalyses

Ocean heat gain

Ocean heat loss

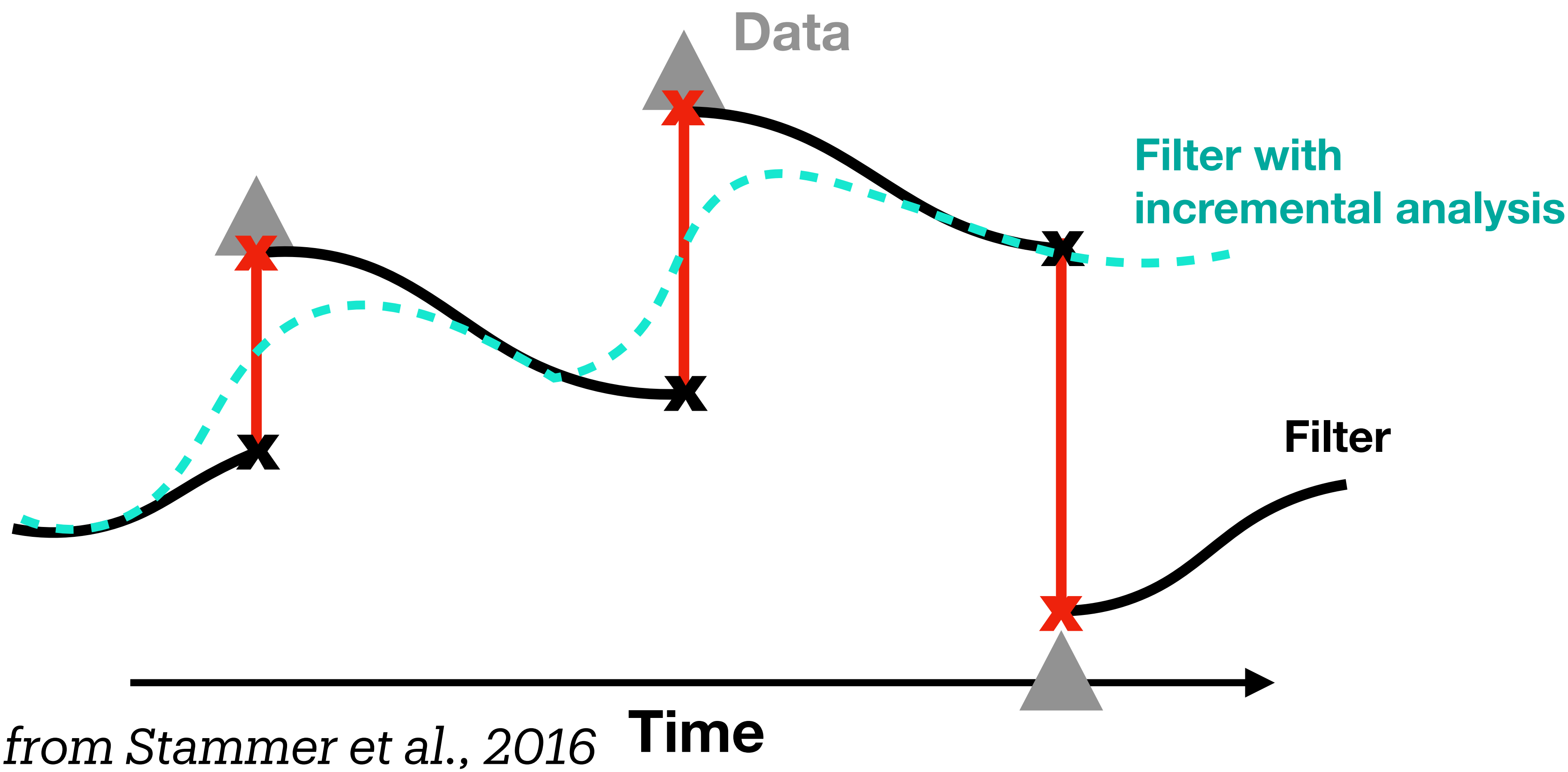


Most ocean reanalyses show spuriously large surface energy imbalance (1993-2009 mean) inconsistent with observed ocean warming (< 1 Wm^{-2}, Roemmich et al., 2015) leading to large unphysical heat removal by the analysis increments

Valdivieso et al., (2015), Balmaseda et al., (2015)

Assessing Budgets: Challenges using reanalyses

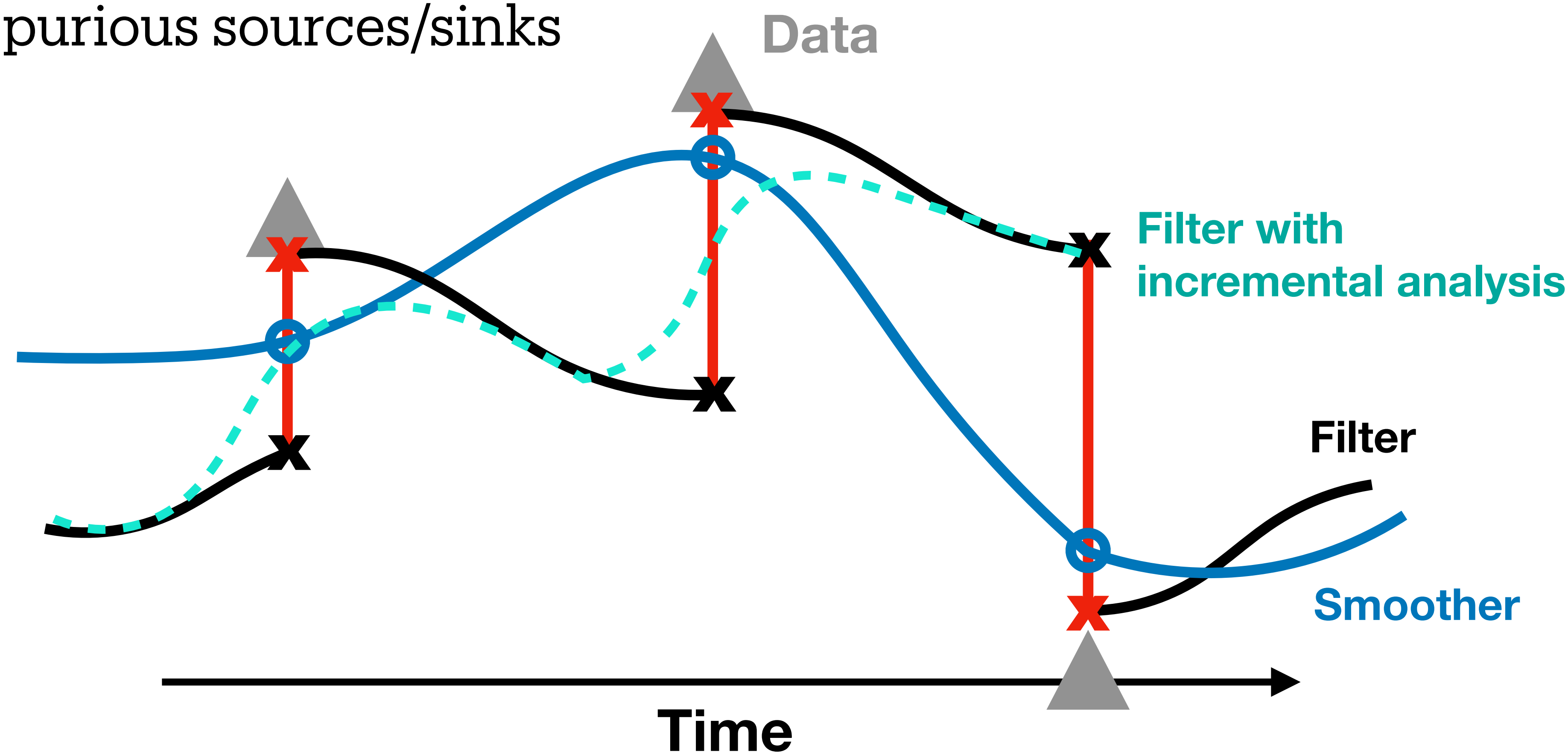
- Target = optimal forecast via IC adjustment
- Dynamical consistency/property conservation *NOT* enforced
- Analysis increments introduce spurious sources/sinks even when transformed to smoothed forcing



Adapted from Stammer et al., 2016

Assessing Budgets: Unique strengths of ECCO

- Target = optimal estimation of entire ocean state over full (multidecadal) period via control adjustment
- Dynamical consistency/property conservation *ENFORCED*
- No spurious sources/sinks

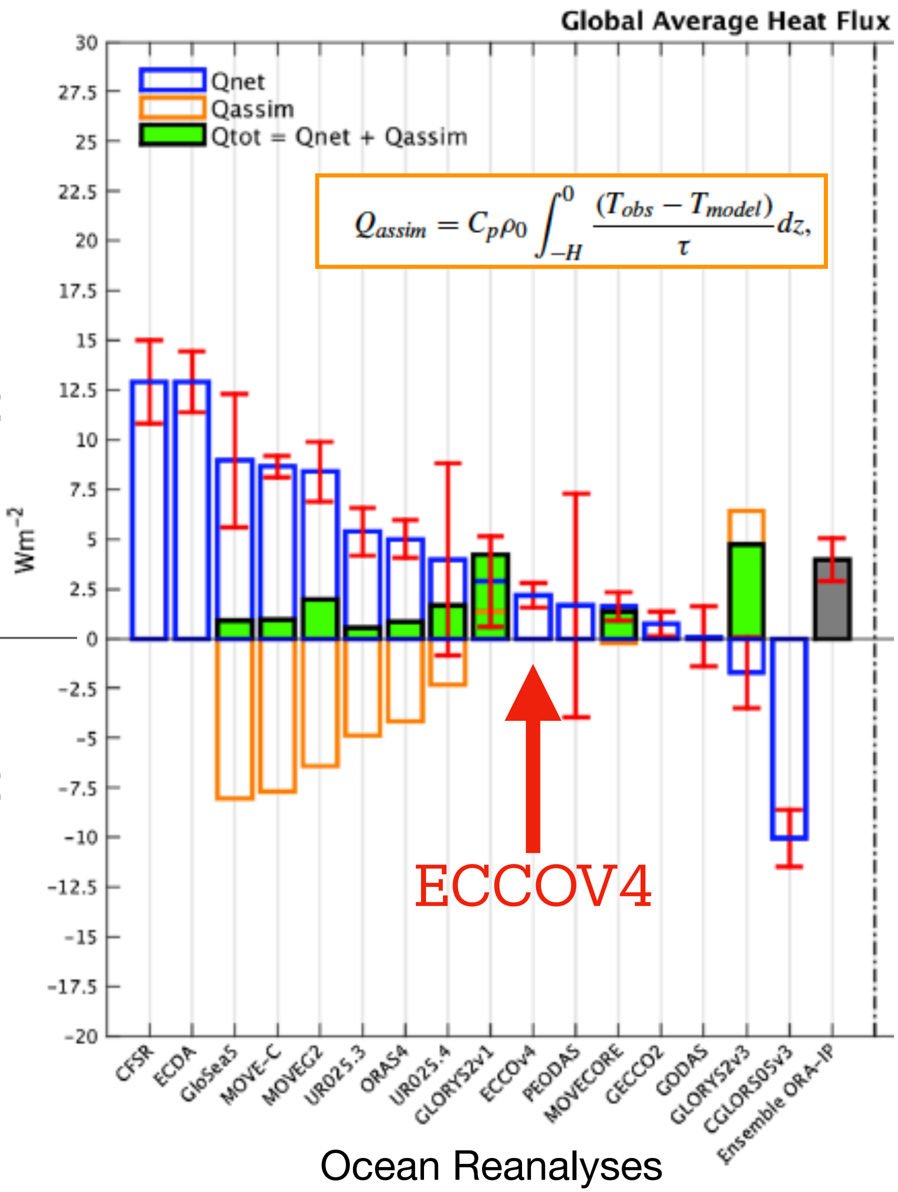


Adapted from Stammer et al., 2016

Assessing Budgets: Challenges using reanalyses

Ocean heat gain

Ocean heat loss



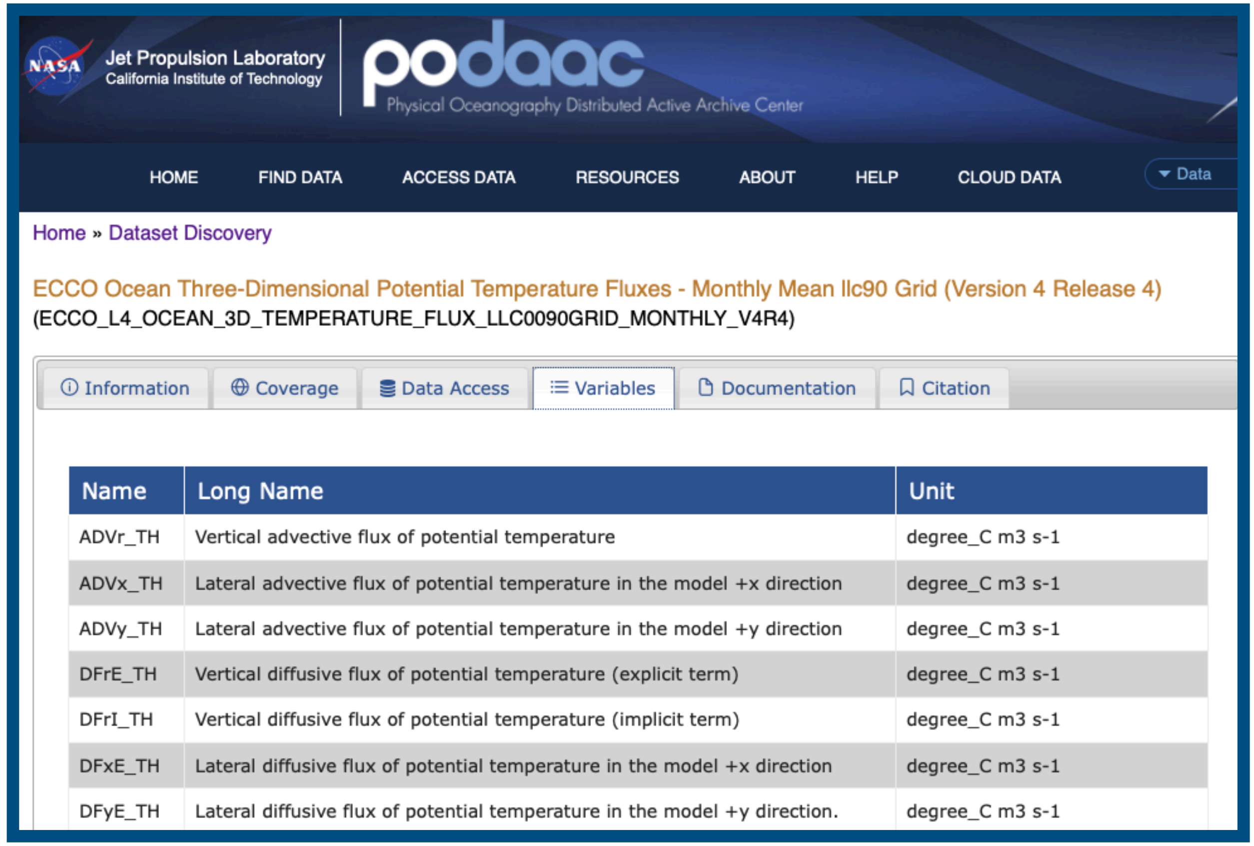
Most ocean reanalyses show spuriously large surface energy imbalance (1993-2009 mean) inconsistent with observed ocean warming (< 1 Wm^{-2}, Roemmich et al., 2015) leading to large unphysical heat removal by the analysis increments

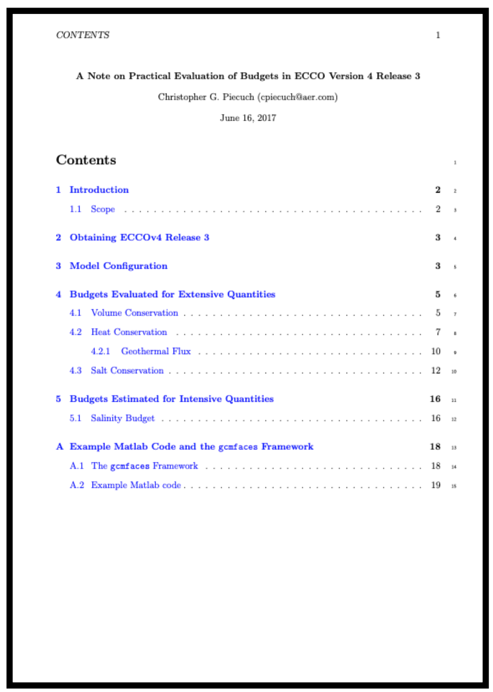
Valdivieso et al., (2015), Balmaseda et al., (2015) Complete global diagnostics

+

accompanying documentation

supports accurate budget assessment





+ ECCO community tutorials!

https://ecco-v4-python-tutorial.readthedocs.io

Detailed Budget Assessments in the ECCO Framework

- **Momentum/vorticity** *e.g., Wunsch 2011, Thomas et al., 2014, Sonnewald et al., 2019, Le Bras et al., 2019, Trossman et al., 2024 …*
- **Sea ice** *e.g., Bigdeli et al., 2019, Schulz et al., 2025*
- **Heat content** *e.g., Buckley et al., 2014, 2015, Tamsitt et al., 2016, Piecuch et al., 2017, Ponte & Piecuch 2018, Asbjørnsen et al., 2020, Tesdal & Haine 2020, Tesdal & Abernathy 2021 …*
- **Salt/salinity/freshwater content** *e.g., Vinogradova & Ponte 2013, Gao et al, 2014, Ponte & Vinogradova 2017, Tesdal & Haine 2020, Verdy et al., 2023, Siddiqui et al., 2024 …*
- **Sea level** *e.g., Piecuch & Ponte 2011, 2012, 2013, 2014 …*
- …

How to assess budgets?

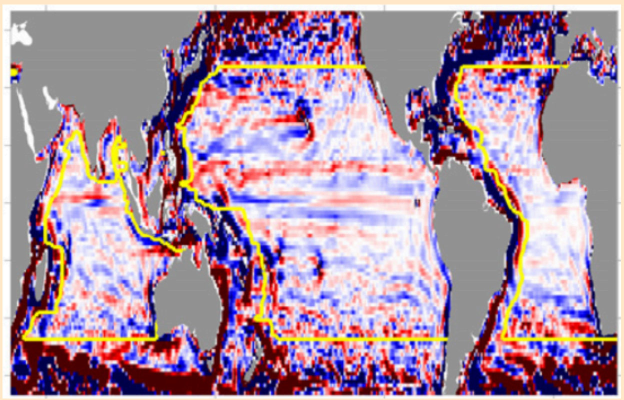
🏆 ECCO state estimates 🏆

✓ Observationally constrained
✓ Dynamically consistent
✓ Complete global diagnostics available
✓ Documentation & detailed tutorials available
✓ A wealth of published applications available

Example Applications

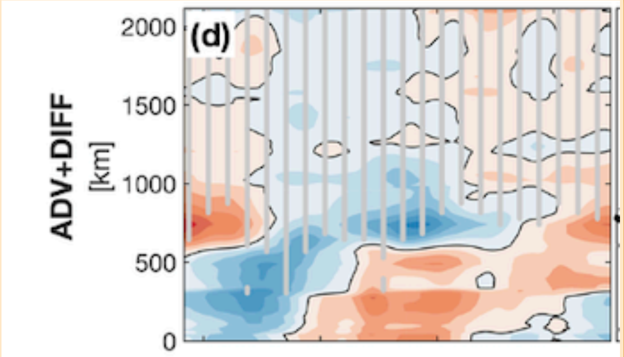
#1 Momentum/Vorticity

On what scales does Sverdrup balance hold?



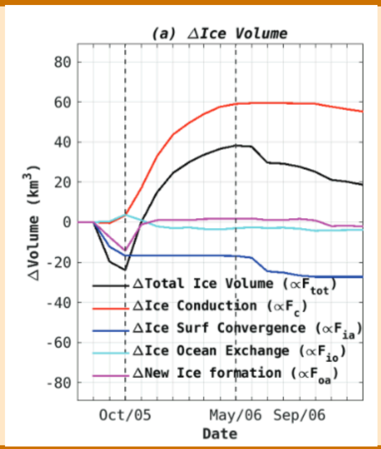
#2 Heat Content

What processes drive Arctic Amplification?



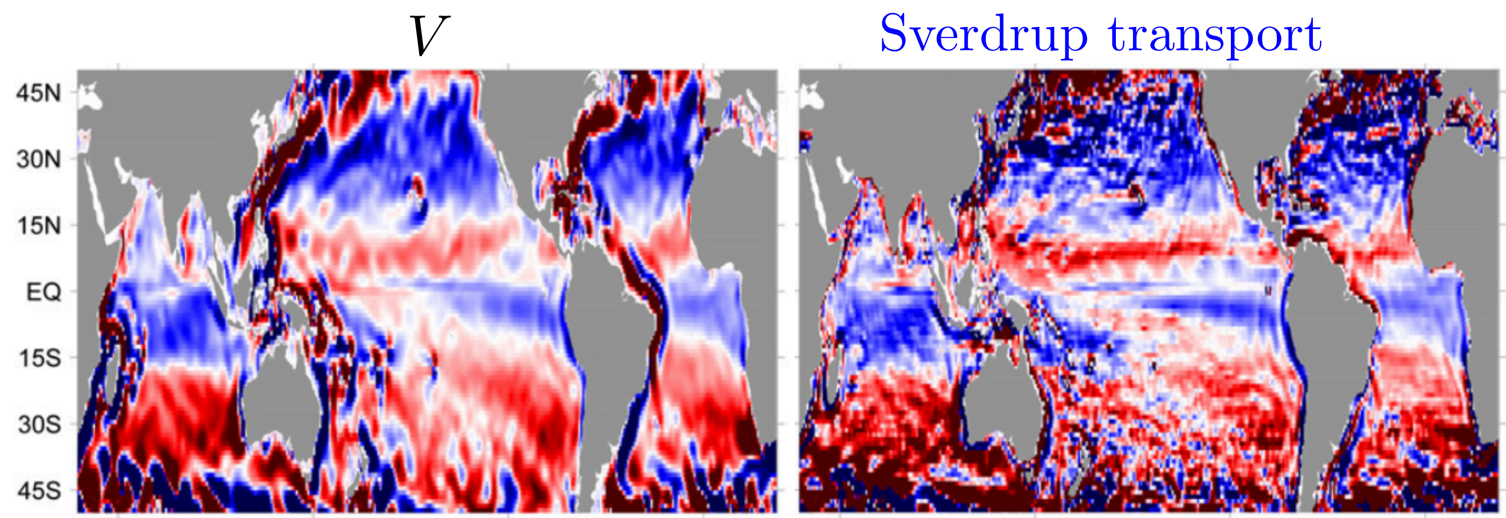
#3 Sea ice volume

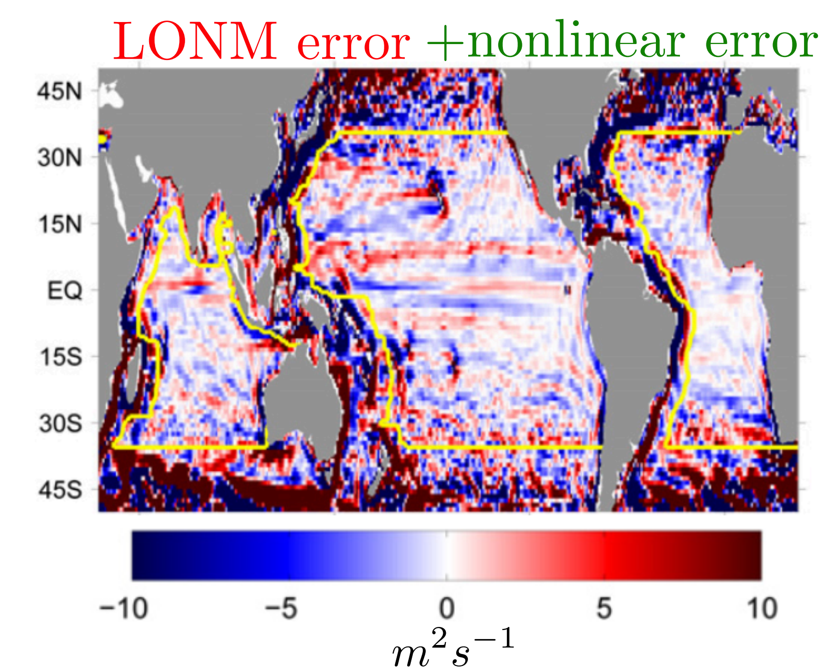
How can atmospheric warming drive sea ice growth?



#1 Testing scales of Sverdrup balance

$$V = (1/\rho_0\beta)\hat{\mathbf{k}}\cdot\nabla\times\tau_{\mathbf{s}} - (f/\beta)w_h + (1/\beta)\hat{\mathbf{k}}\cdot\nabla\times\int_{-h}^{s}\nabla\cdot(A_H\nabla\mathbf{u}_H)dz - (1/\beta)\hat{\mathbf{k}}\cdot\nabla\times\int_{-h}^{s}(\mathbf{u}\cdot\nabla)\mathbf{u}\,dz$$





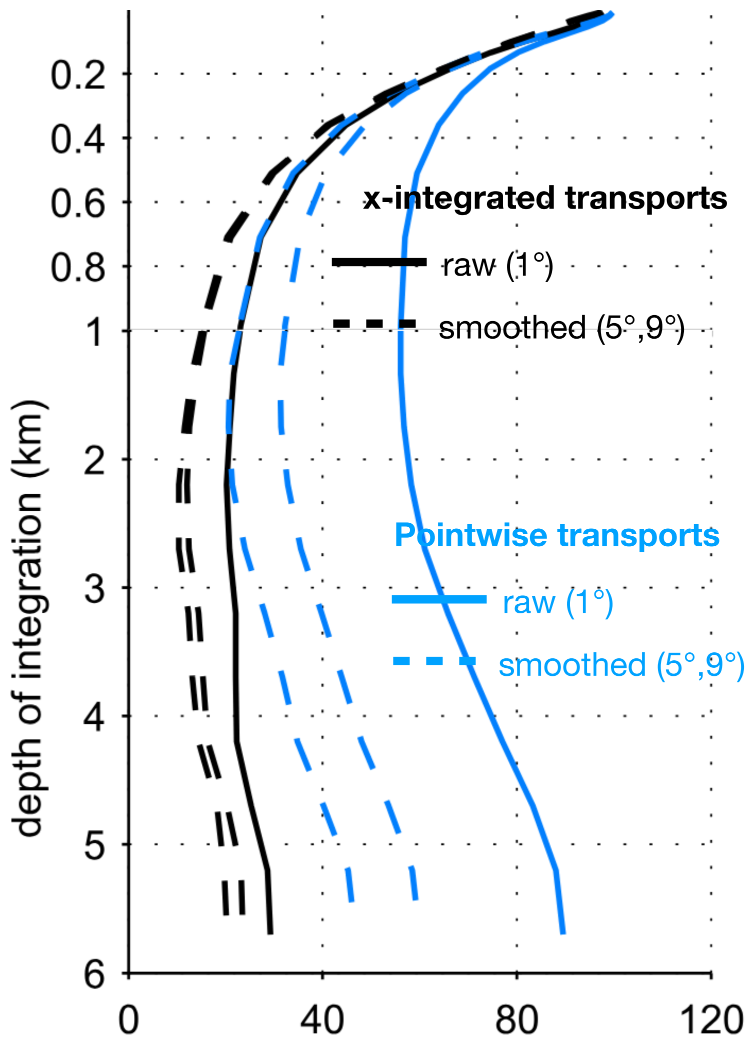
ECCOV4r3
1992-2007 mean;
assume h=1400m

Thomas et al., 2014

#1 Testing scales of Sverdrup balance

Thomas et al., 2014

$$\text{error (\%)} = 100 \times \frac{\text{LONM error} + \text{nonlinear error}}{\text{Sverdrup transport}}$$



Summary:

Sverdrup balance holds (error < 30%) in subtropical interior only:

(1) When integrating to mid-depth (below main thermocline, above deep transports)

(2) At scales > 5°

Identifying dynamically distinct circulation regimes

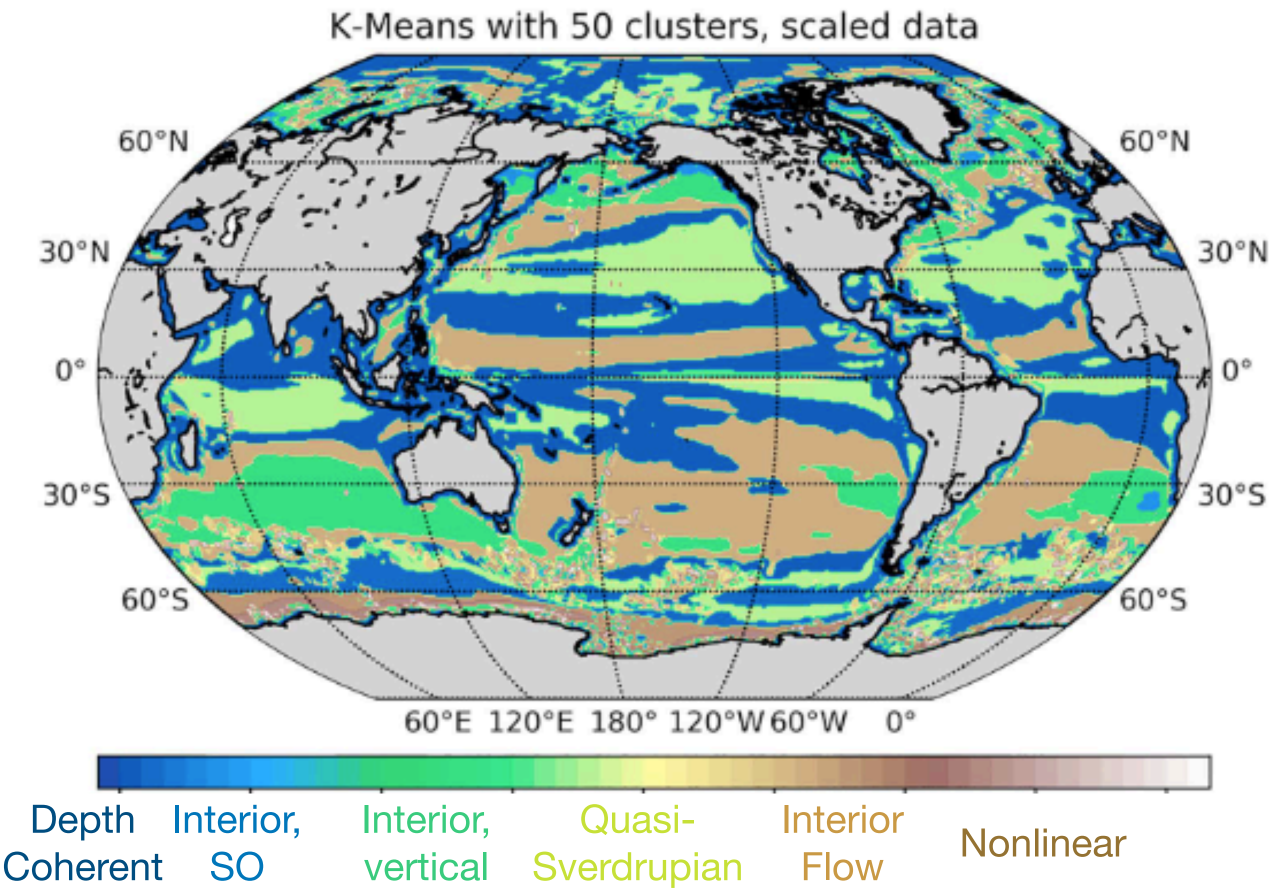


Table 1

Percentage of Area Covered by the Area-Specific Balance of the BV Equation (3) and the Corresponding Map Figure

Cluster	Area	Leading terms
1	43 ± 3.3%, Depth coherent (Figure 3a)	$\nabla \times \tau_{sb} + \nabla \times \mathbf{A} \approx \nabla p_b \times \nabla H$ (Figure 3b)
2	24.8 ± 1.2%, Interior flow (Figure 3c)	$\nabla \times \tau_{sb} \approx \nabla p_b \times \nabla H + \nabla \cdot (f\mathbf{U})$ (Figure 3d)
3	14.6 ± 1%, Quasi-Sverdrupian (Figure 3e)	$\nabla \times \tau_{sb} \approx \nabla \cdot (f\mathbf{U})$ (Figure 3f)
4	6.9 ± 2.9%, Interior flow, vertical (Figure 4a)	$\nabla \times \tau_{sb} \approx \nabla \cdot (f\mathbf{U}) + \nabla p_b \times \nabla H$ (Figure 4b)
5	1.9 ± 1%, Interior flow, Southern Ocean (Figure 4c)	$\nabla \times \tau_{sb} \approx \nabla \cdot (f\mathbf{U}) + \nabla p_b \times \nabla H$ (Figure 4d)
6–50	8.9 ± 0.3%, Dominantly nonlinear (Figure 4e)	$\nabla \cdot (f\mathbf{U}) \approx \nabla \times \mathbf{A} + \nabla \times \tau_{sb}$ (Figure 4f)

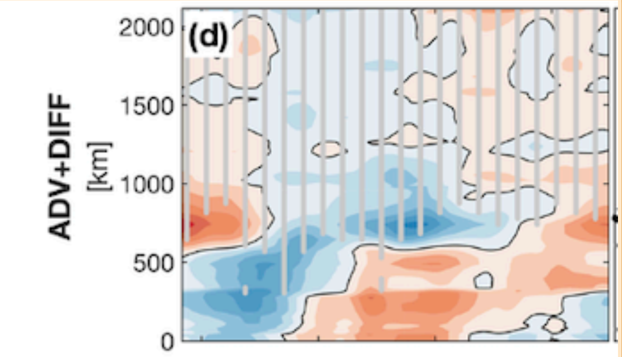
Note. Leading order terms are sorted by magnitude, colors indicating if barotropic vorticity is added (red in font) or removed (blue in font) by the leading order term, the corresponding bar chart figure shows the full breakdown. The quoted percentage coverage and StD is the mean of 100 runs of the algorithm.

Sonnewald et al., 2019

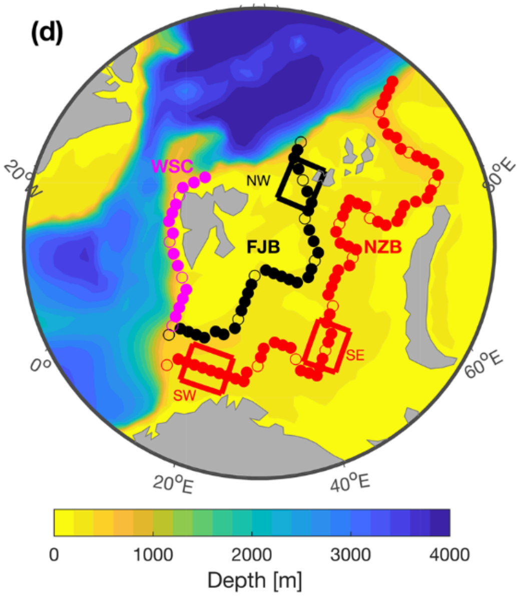
Example Applications

#2 Heat Content

What processes drive Arctic Amplification?

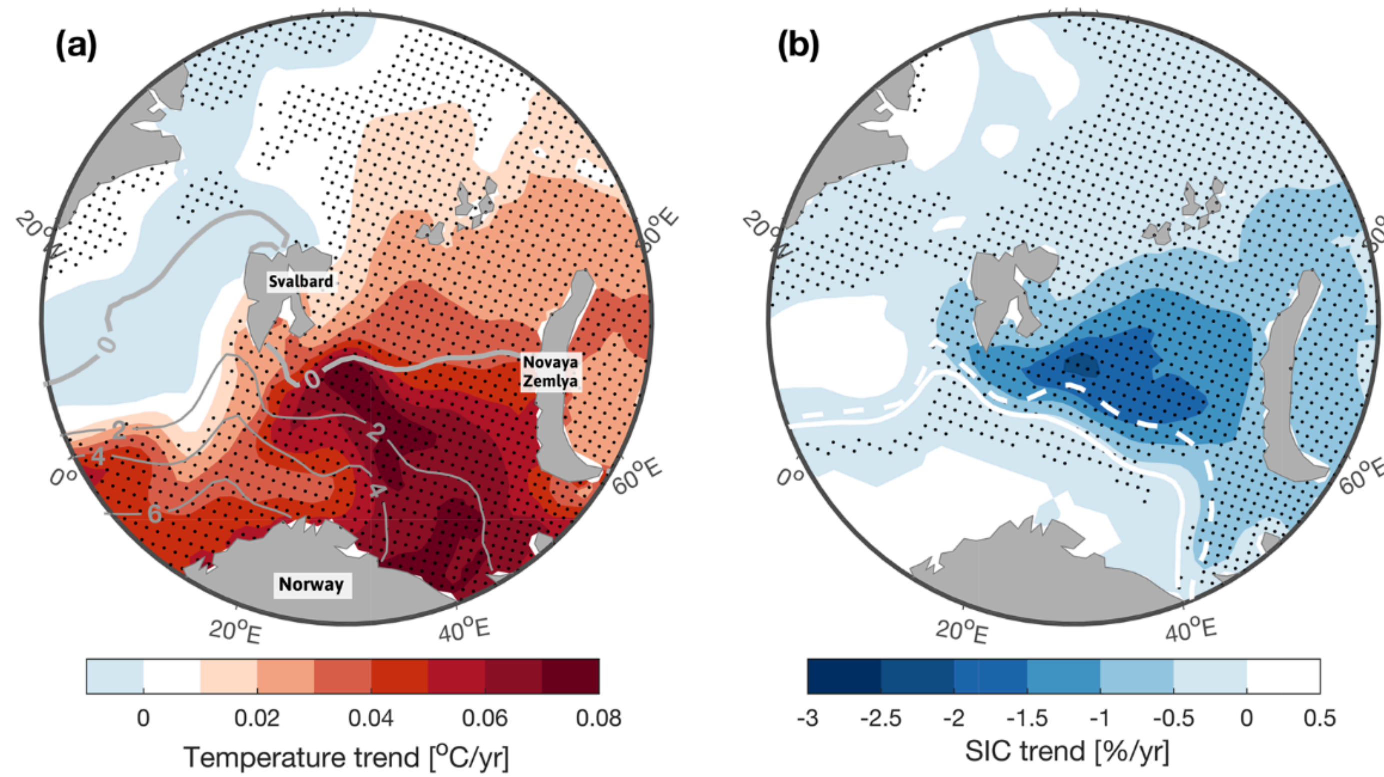


#2 Exploring mechanisms of Arctic Amplification



Mean AW pathways through the Barents Sea

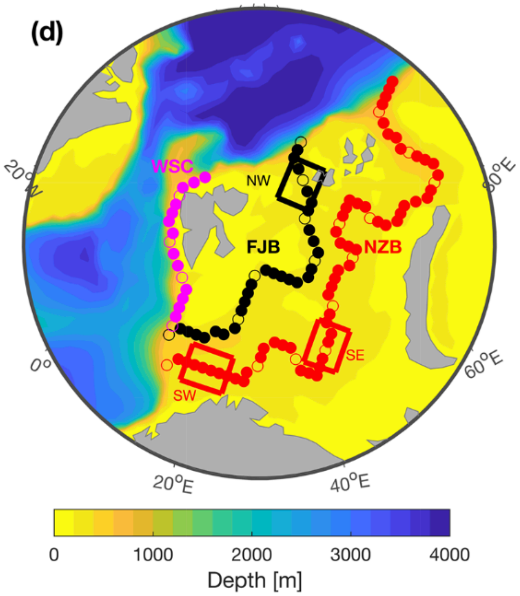
ECCOV4r3 trends 1993-2014



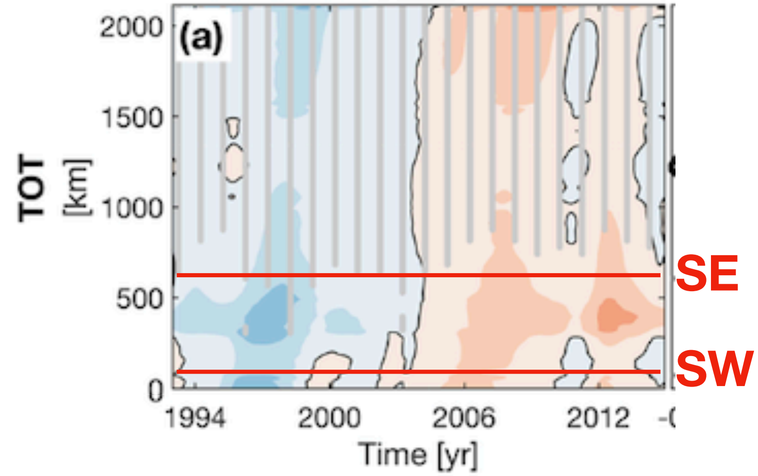
Asbjørnsen et al., 2020

#2 Exploring mechanisms of Arctic Amplification

Asbjørnsen et al., 2020

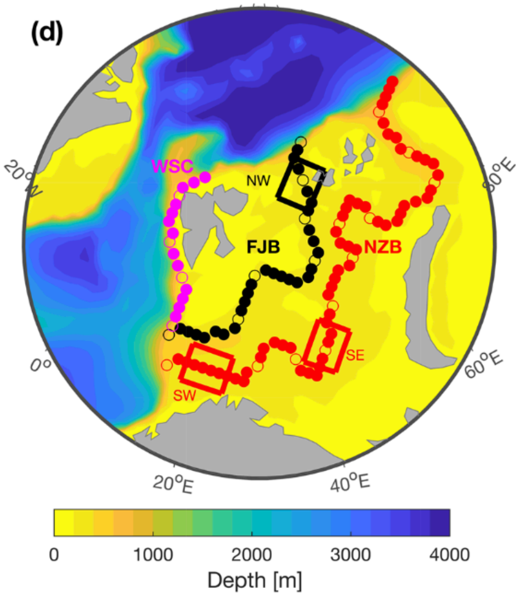


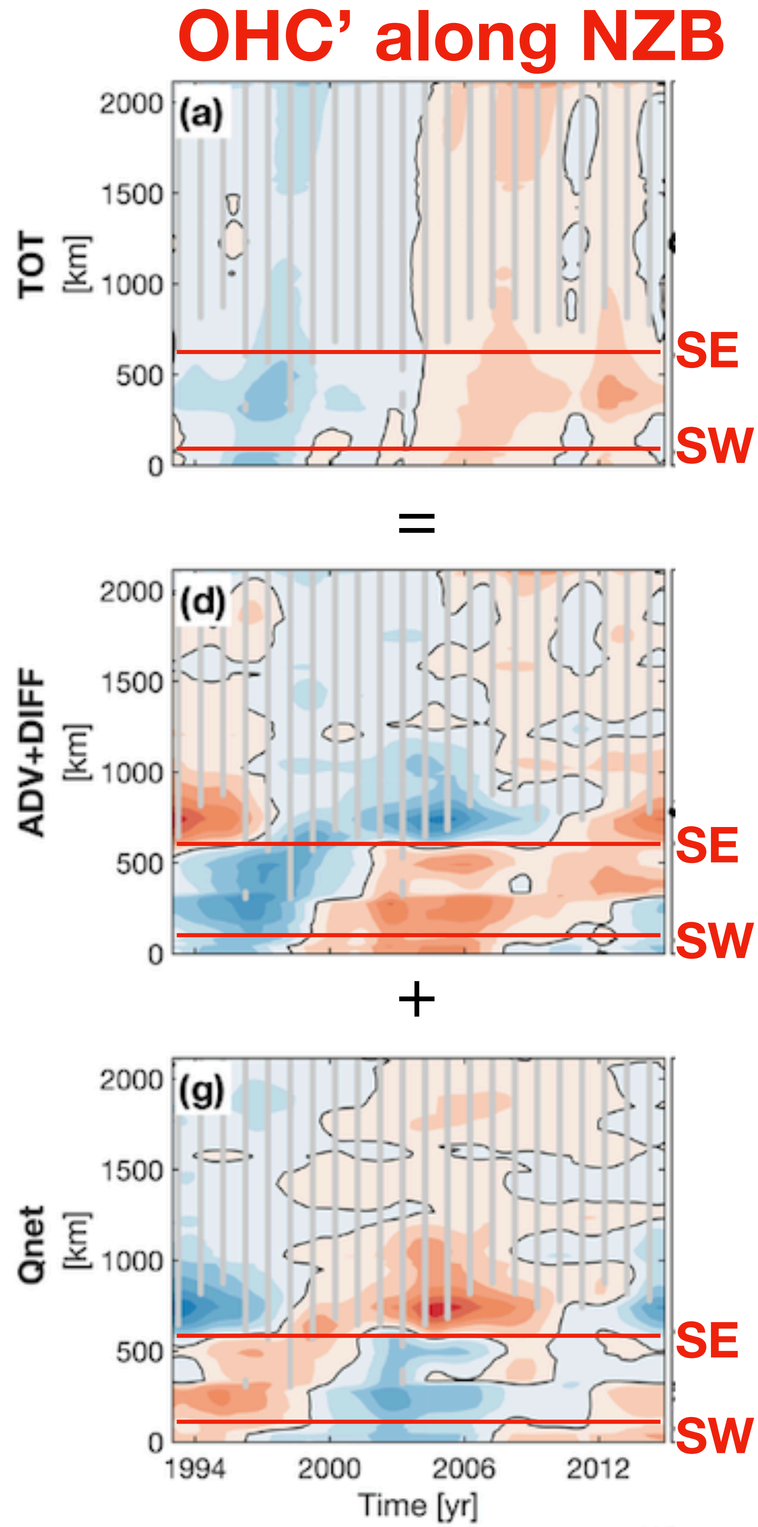
OHC' along NZB



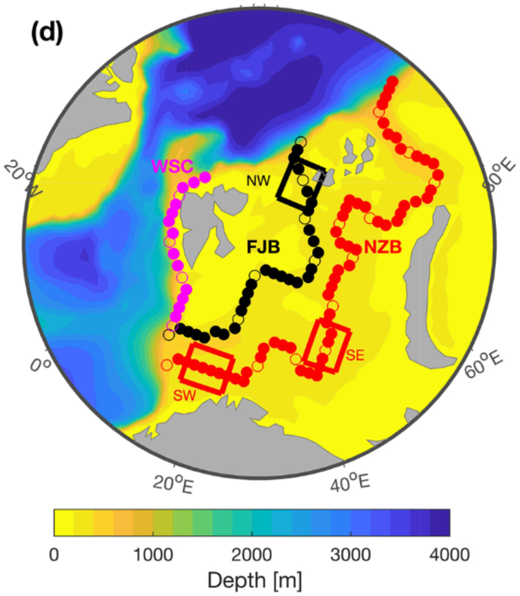
#2 Exploring mechanisms of Arctic Amplification

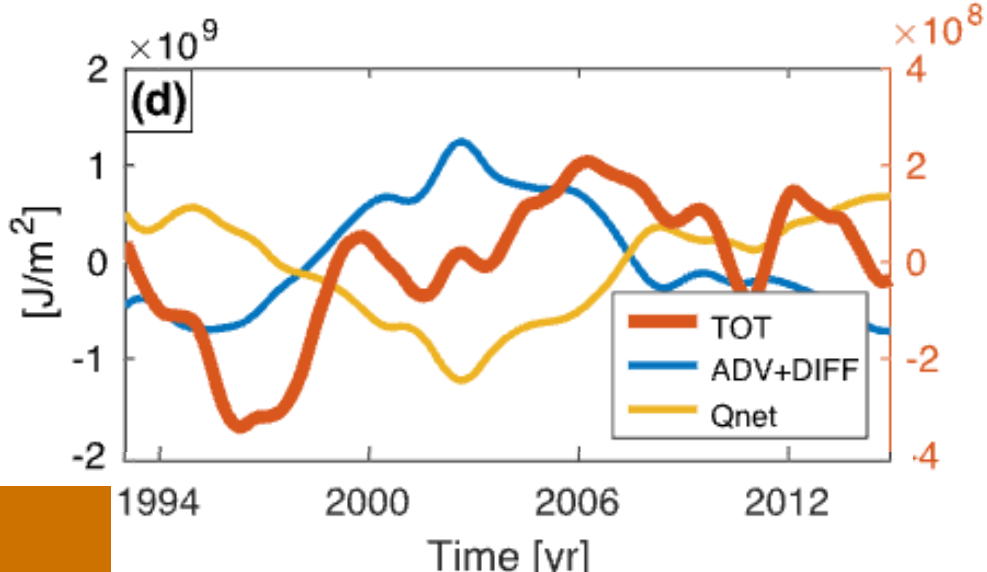
Asbjørnsen et al., 2020

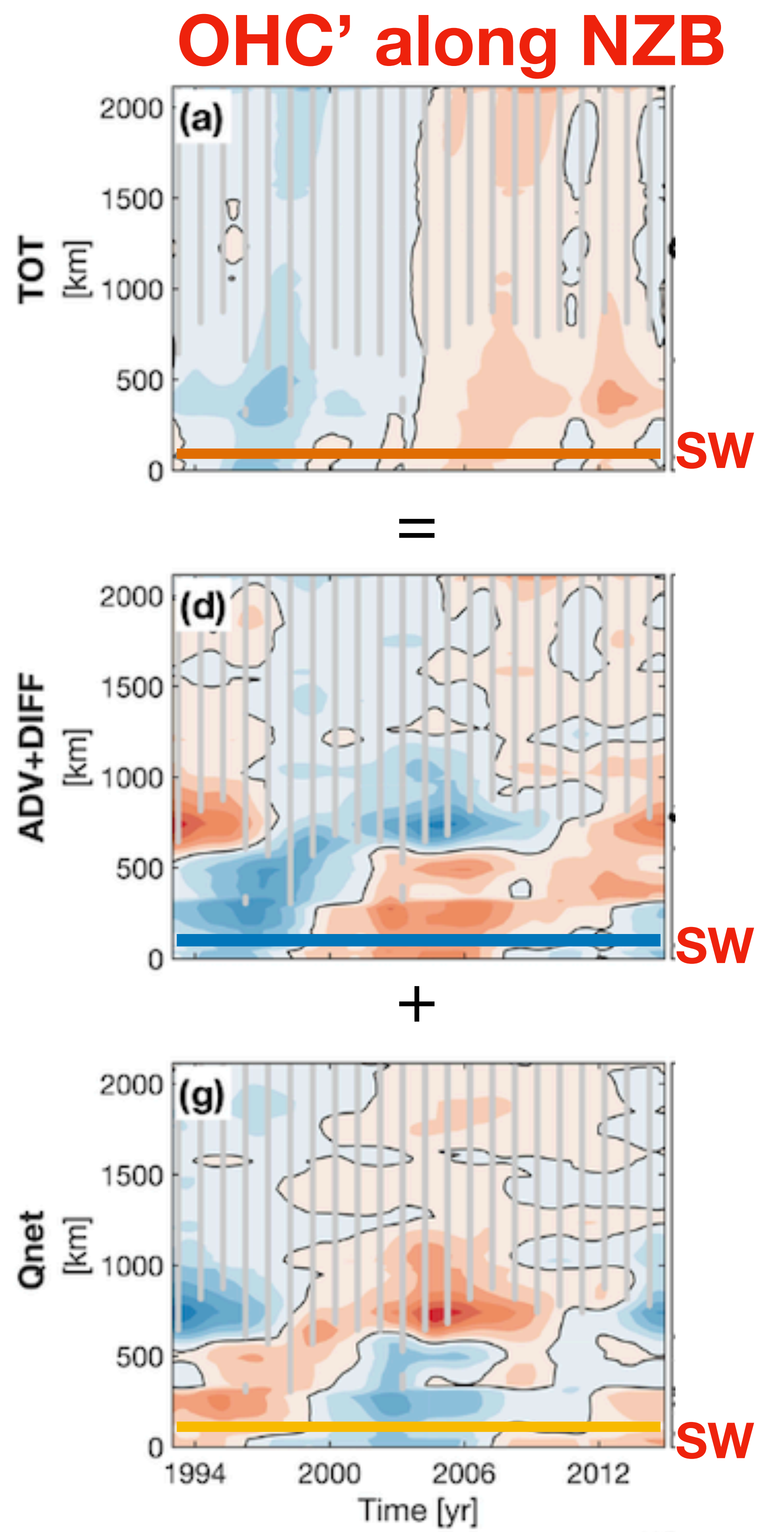




#2 Exploring mechanisms of Arctic Amplification

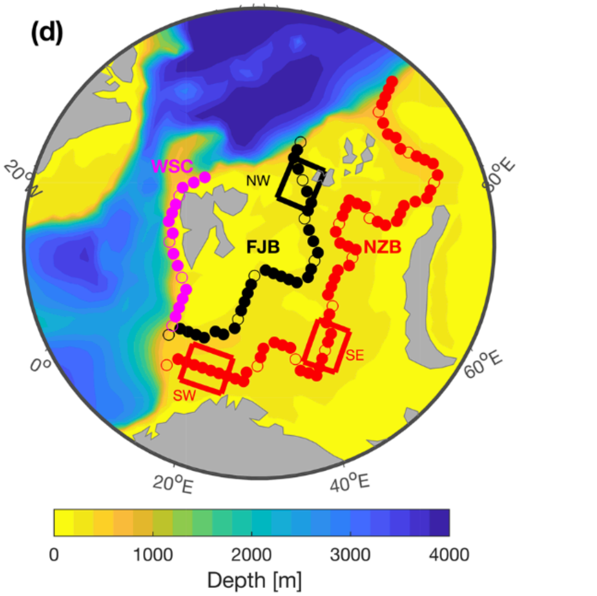


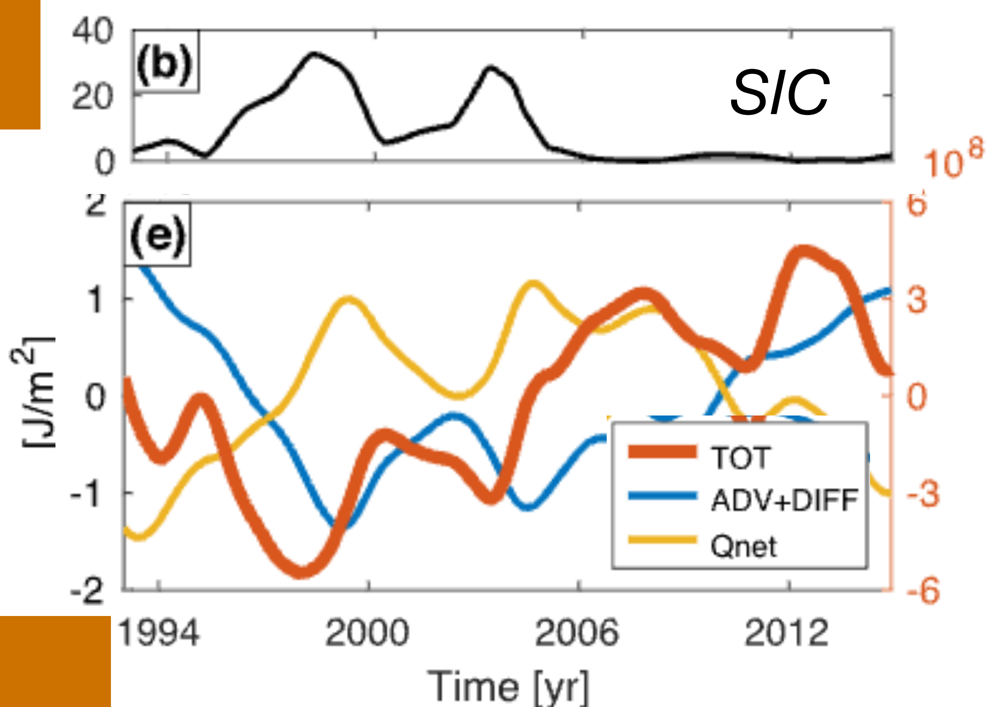




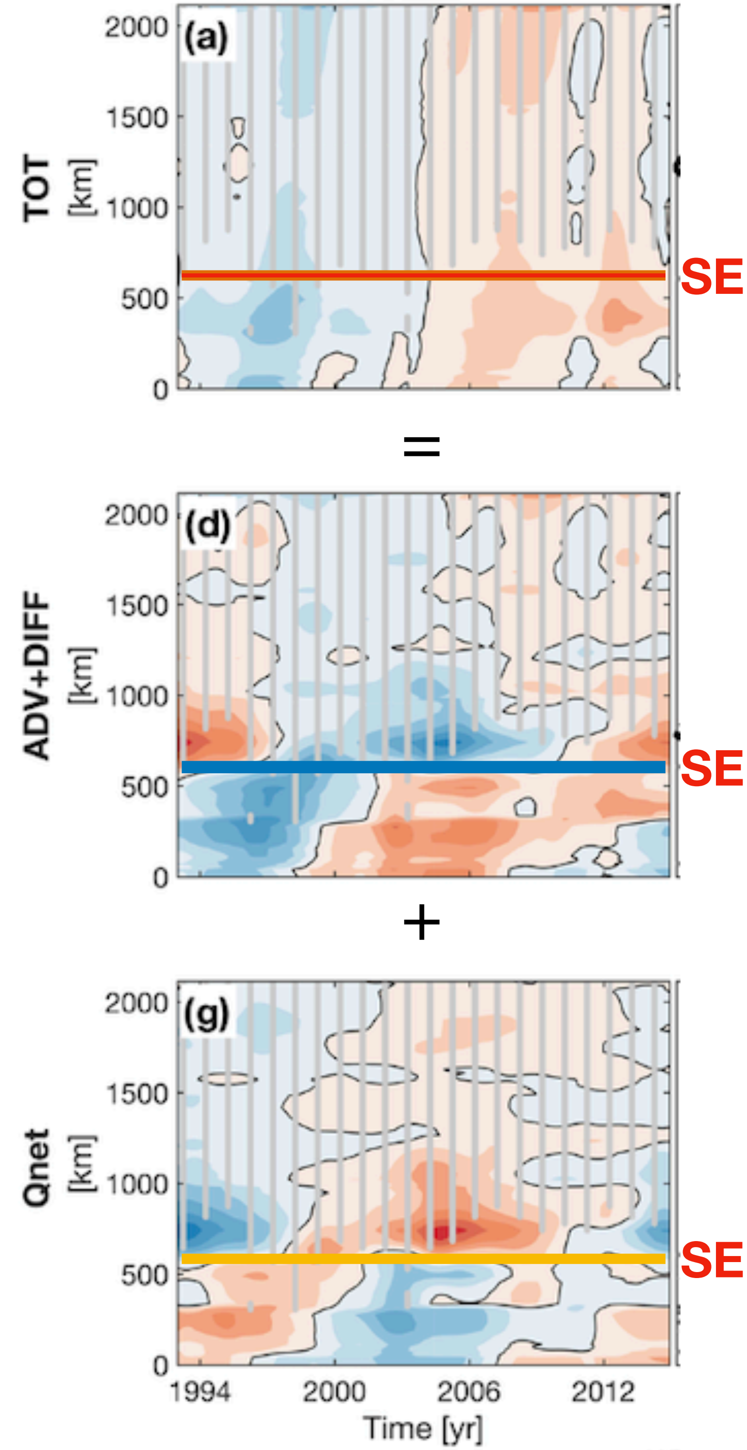
Asbjørnsen et al., 2020

#2 Exploring mechanisms of Arctic Amplification



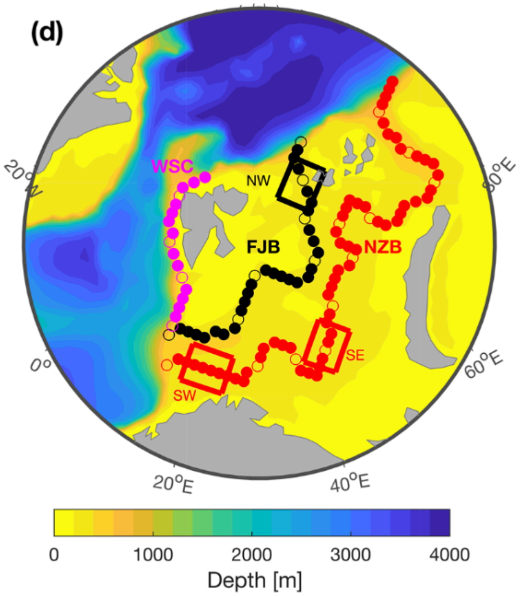


OHC' along NZB

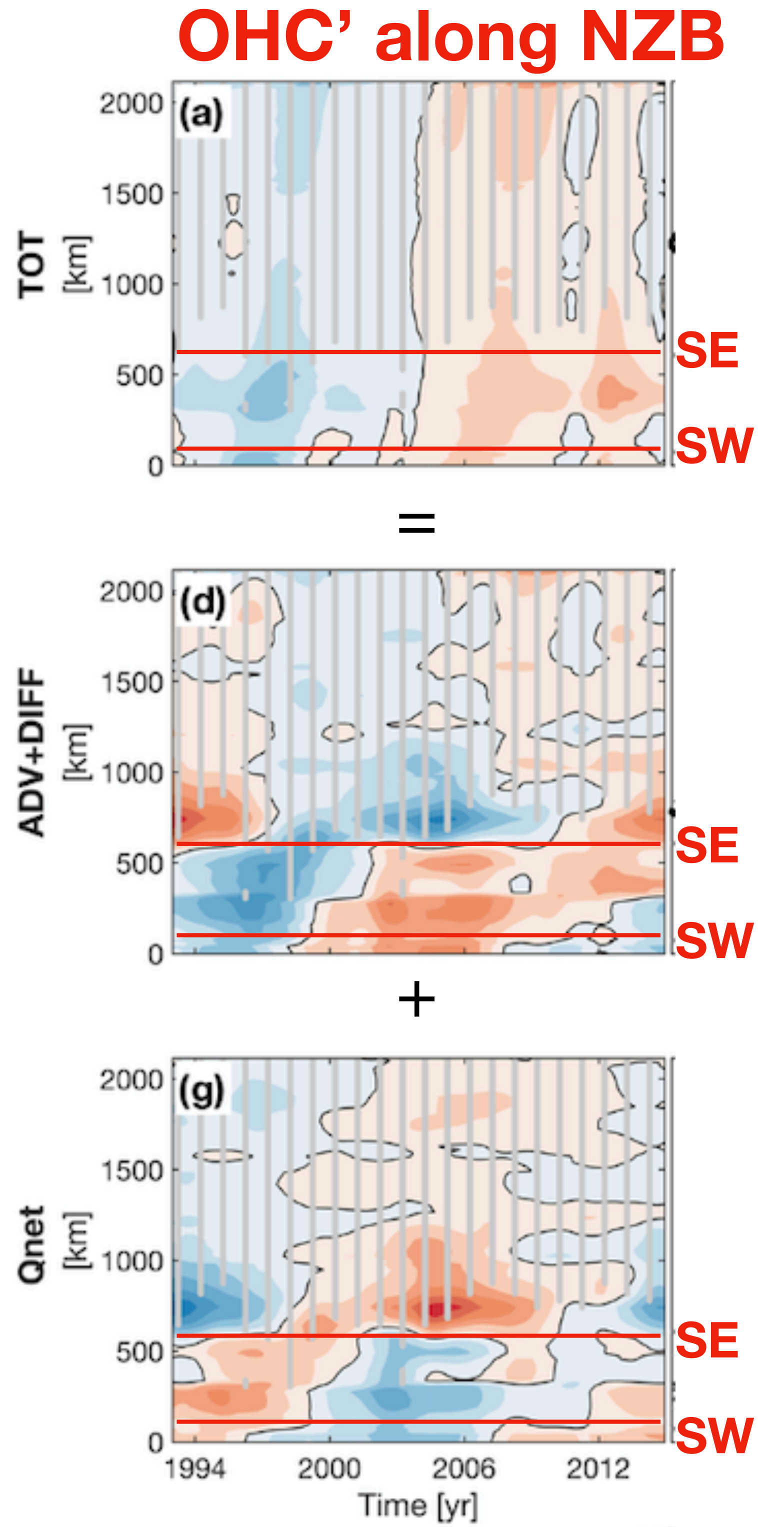


Asbjørnsen et al., 2020

#2 Exploring mechanisms of Arctic Amplification



Summary: *Ongoing Atlantification is complex! Underlying mechanisms vary in space and time*

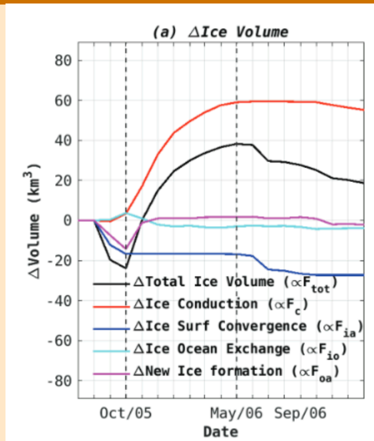


Asbjørnsen et al., 2020

Example Applications

#3 Sea ice volume

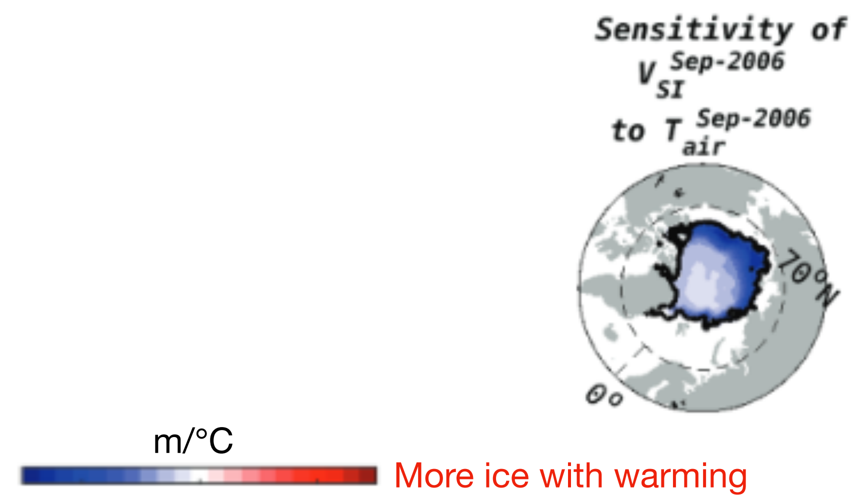
How can atmospheric warming drive sea ice growth?



#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(a) **counterintuitive** sensitivity of ice volume to T_{air}

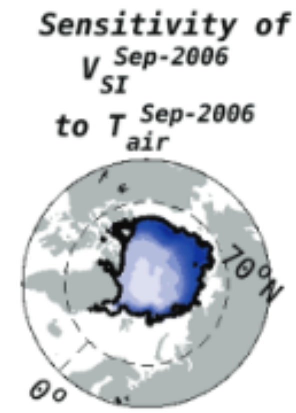


#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(a) **counterintuitive** sensitivity of ice volume to T_{air}

Sensitivity at earlier lead time?



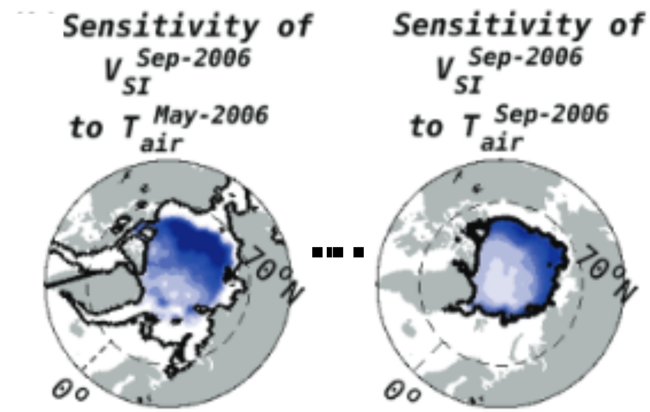
Less ice with warming | m/°C | More ice with warming

#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(a) counterintuitive sensitivity of ice volume to T_{air}

Sensitivity at earlier lead time?

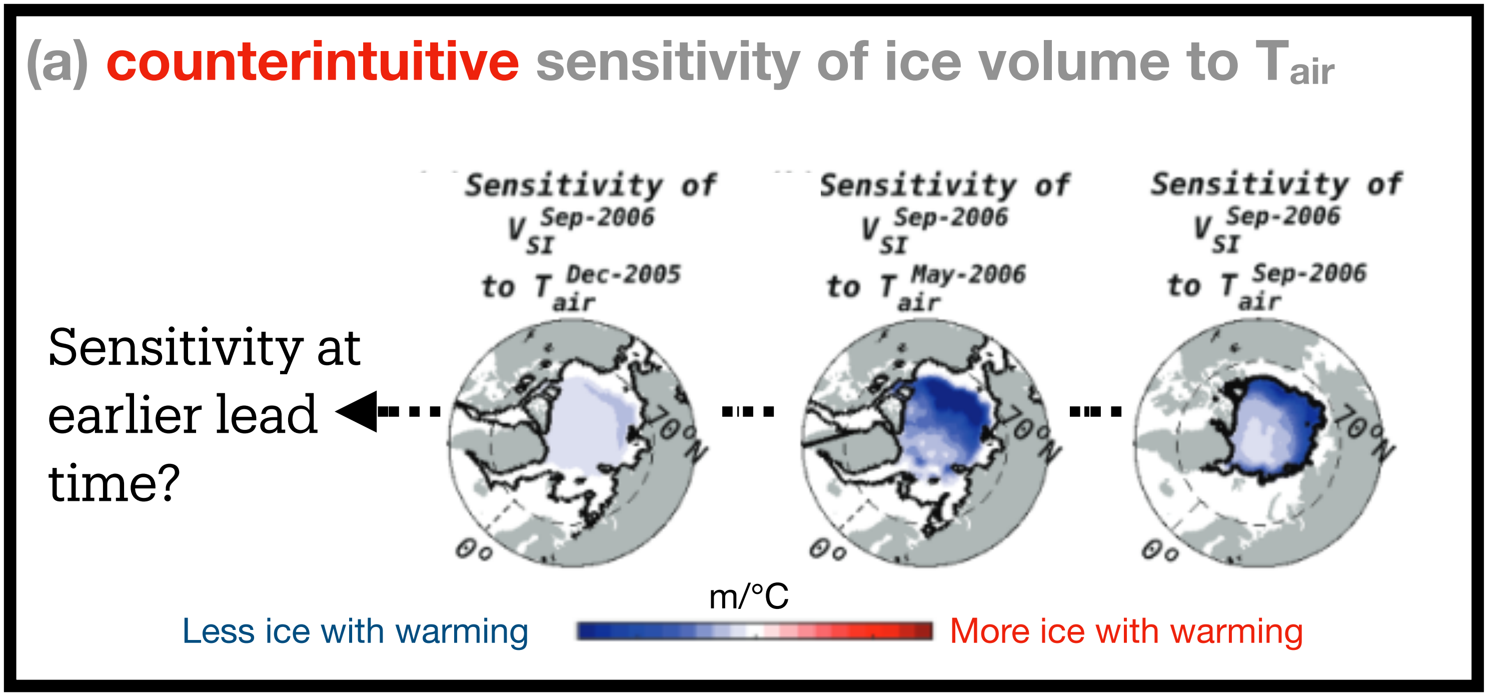


m/°C

Less ice with warming | More ice with warming

#3 Exploring drivers of Arctic sea ice variability

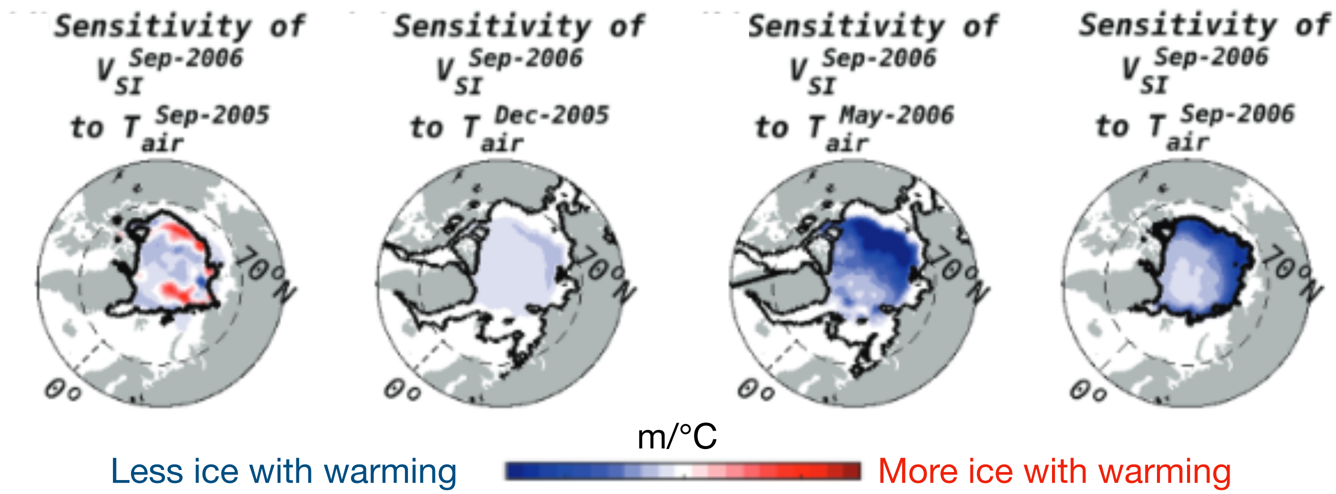
Bigdeli et al., 2020



#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

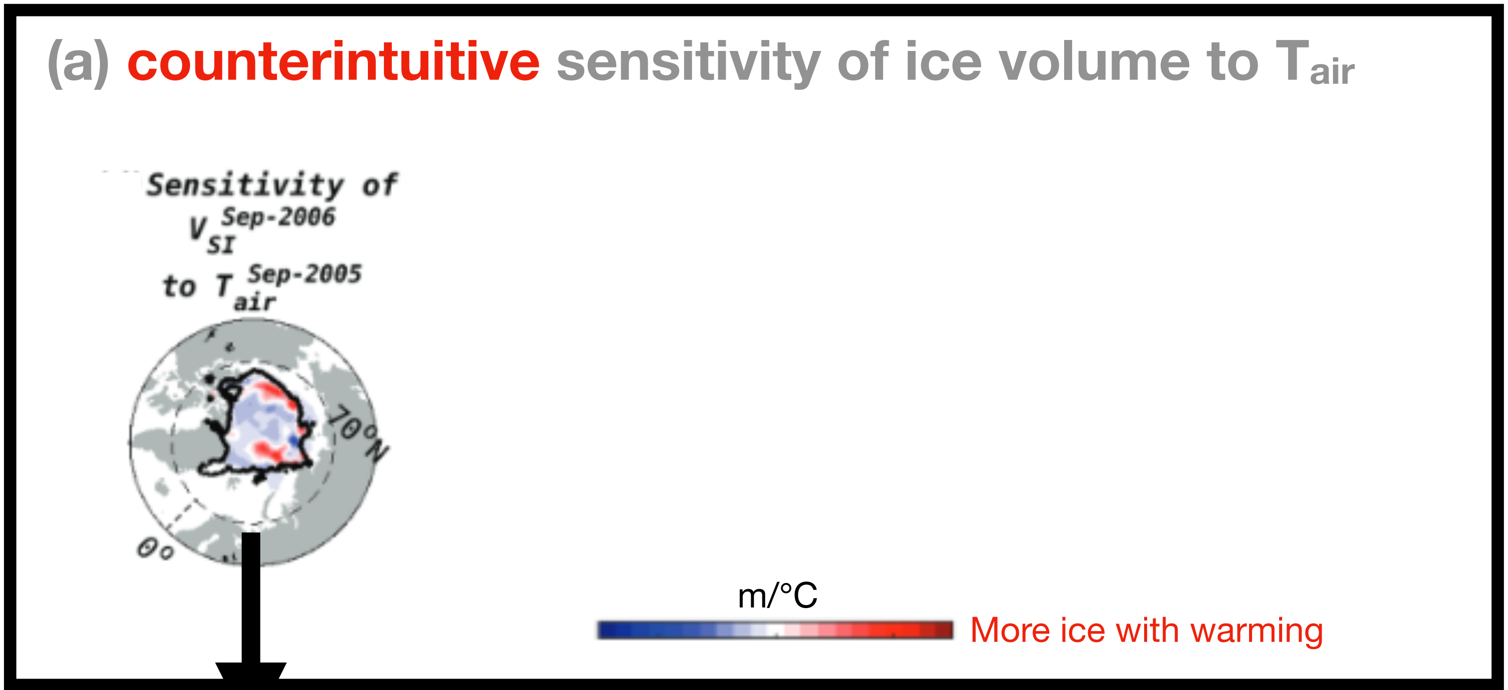
(a) **counterintuitive** sensitivity of ice volume to T_{air}



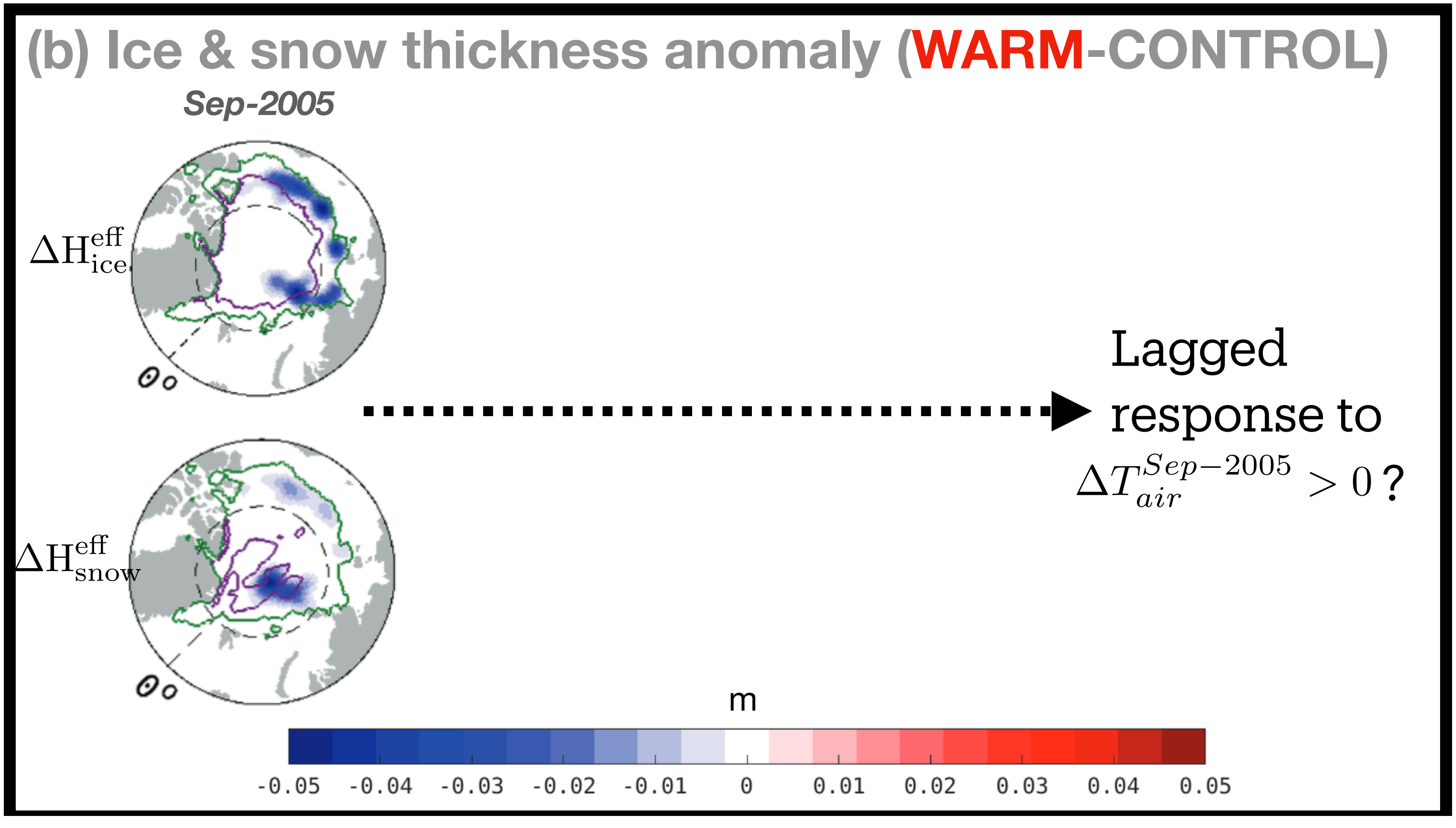
#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(a) **counterintuitive** sensitivity of ice volume to T_{air}



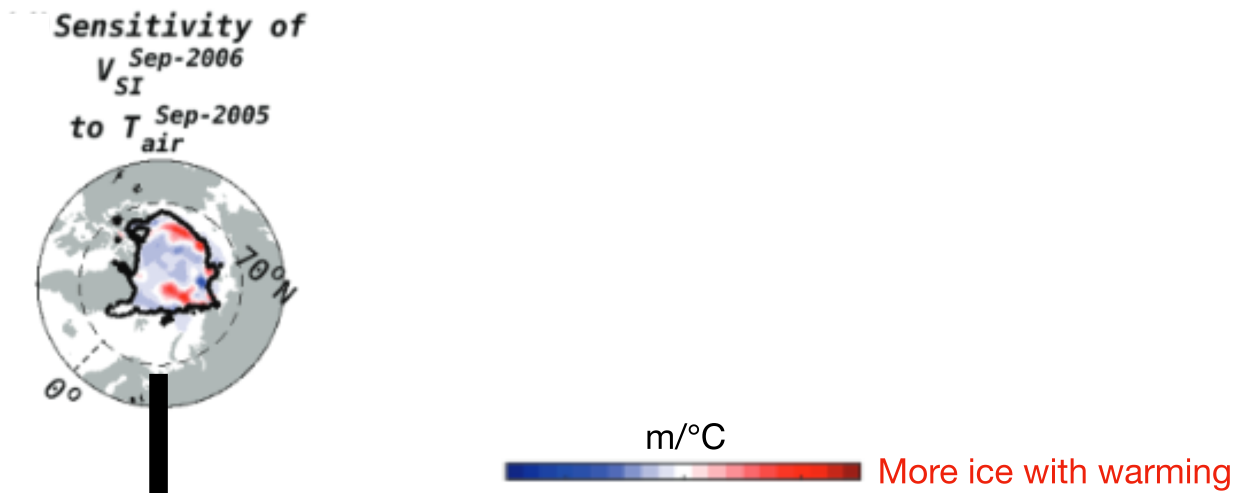
(b) Ice & snow thickness anomaly (**WARM**-CONTROL)



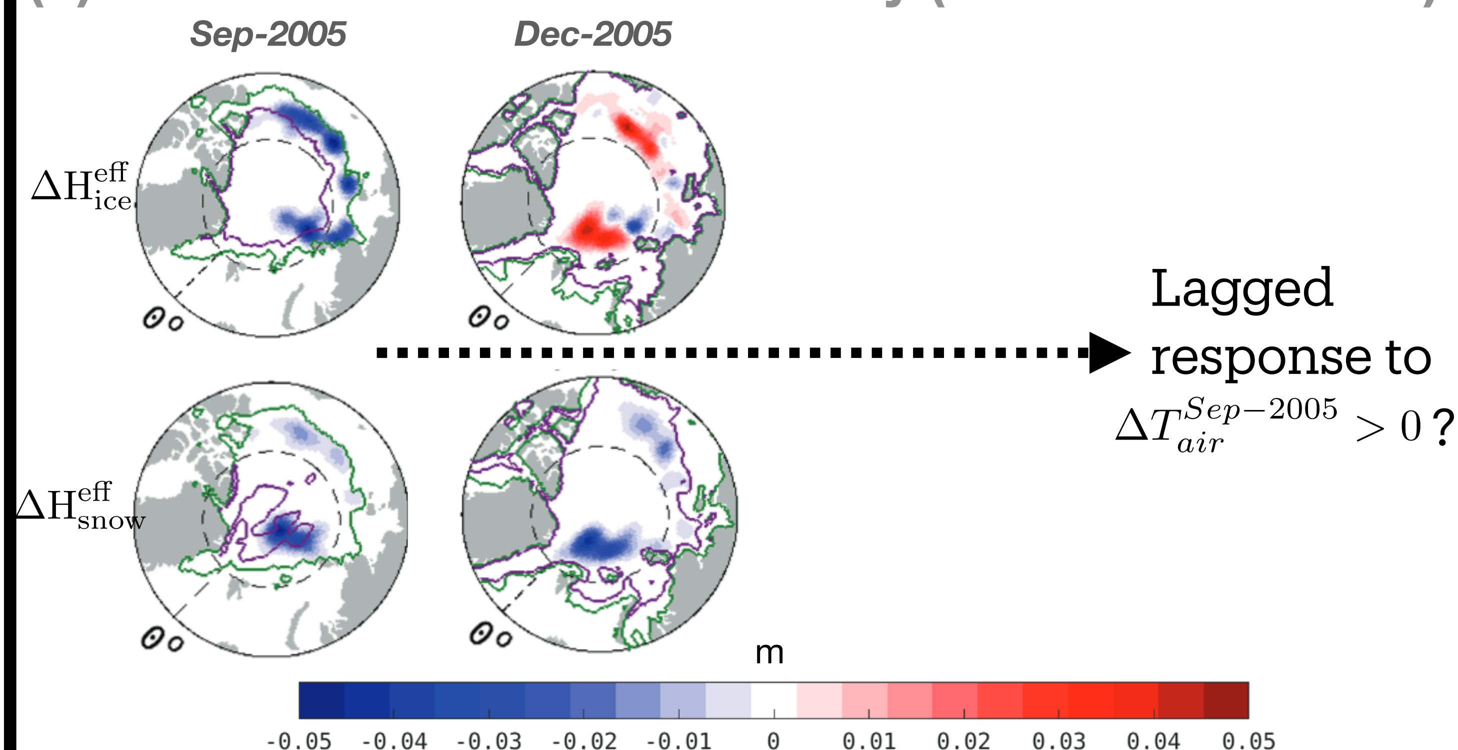
#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

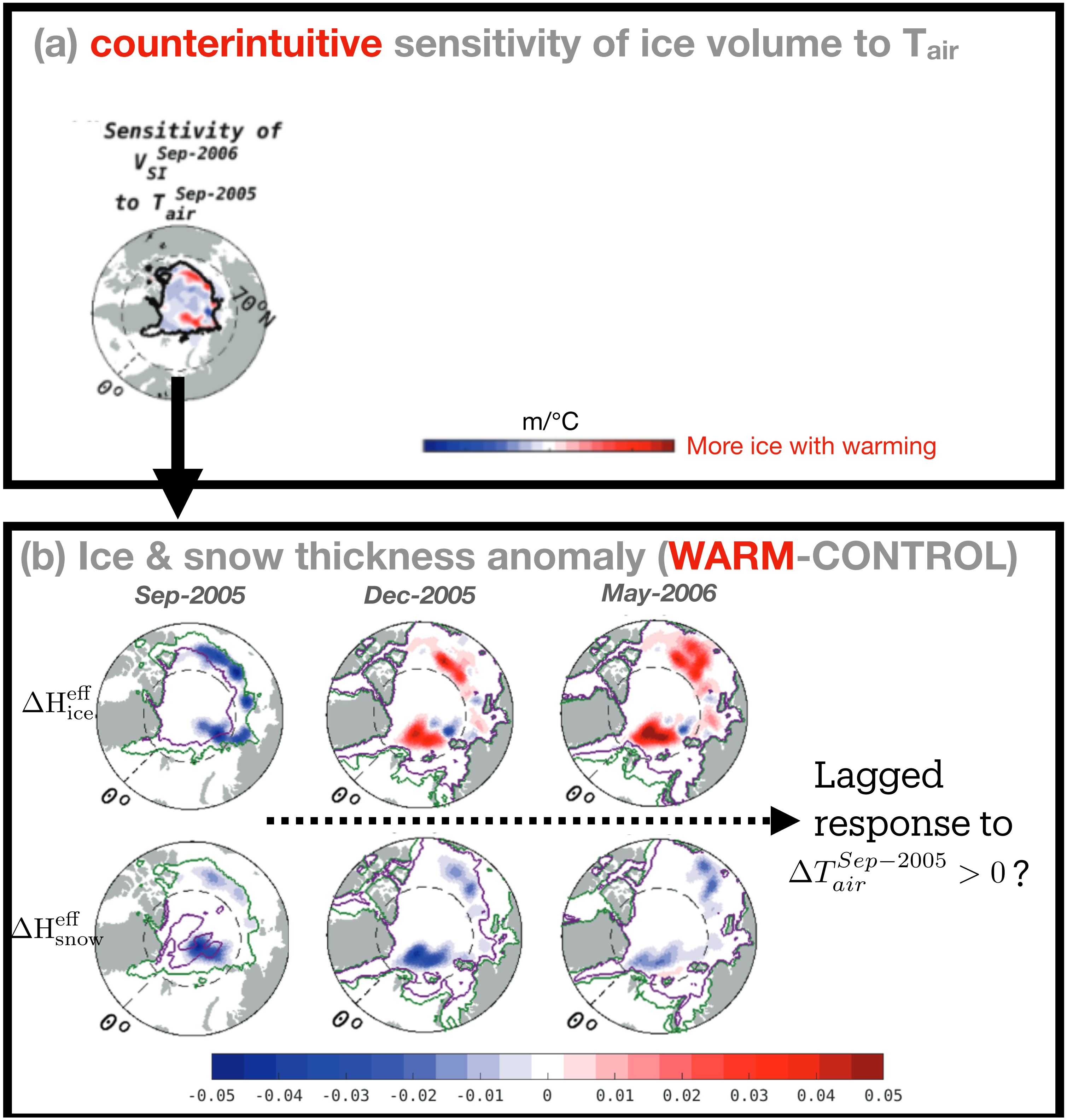
(a) counterintuitive sensitivity of ice volume to T_{air}



(b) Ice & snow thickness anomaly (WARM-CONTROL)



#3
Exploring drivers of Arctic sea ice variability

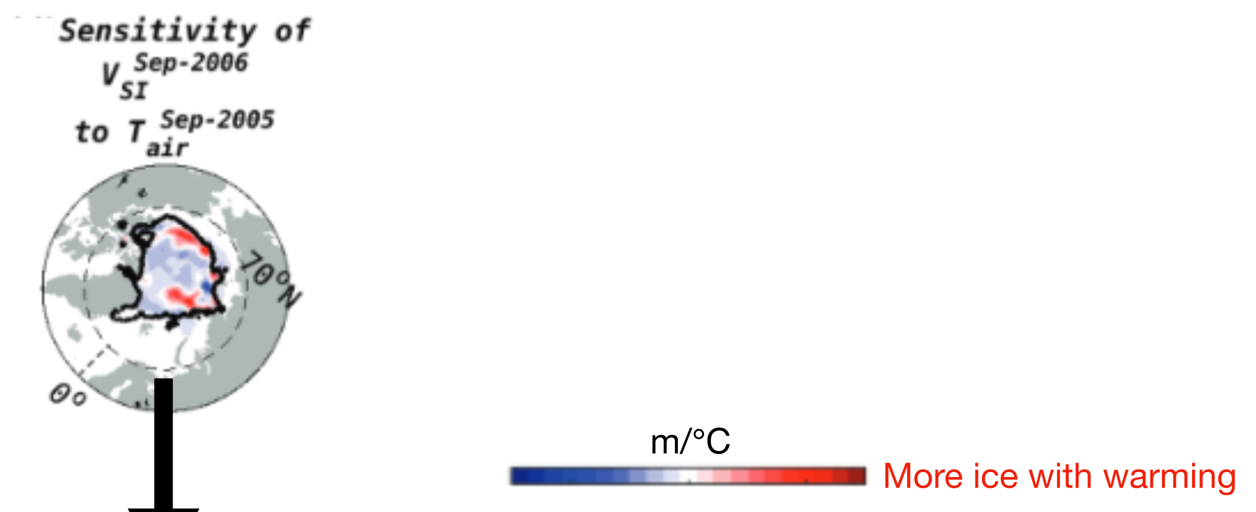


Bigdeli et al., 2020

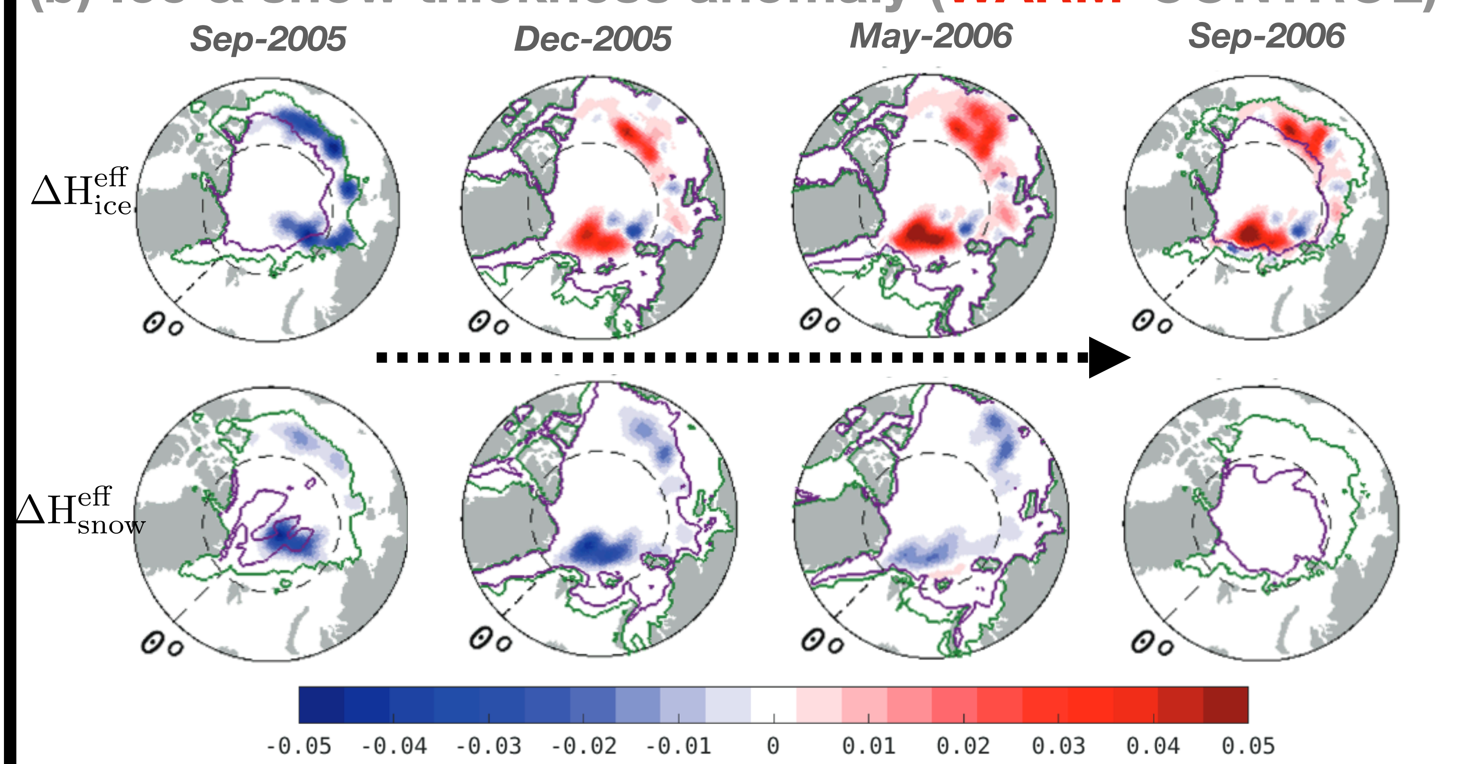
#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(a) counterintuitive sensitivity of ice volume to T_{air}



(b) Ice & snow thickness anomaly (WARM-CONTROL)

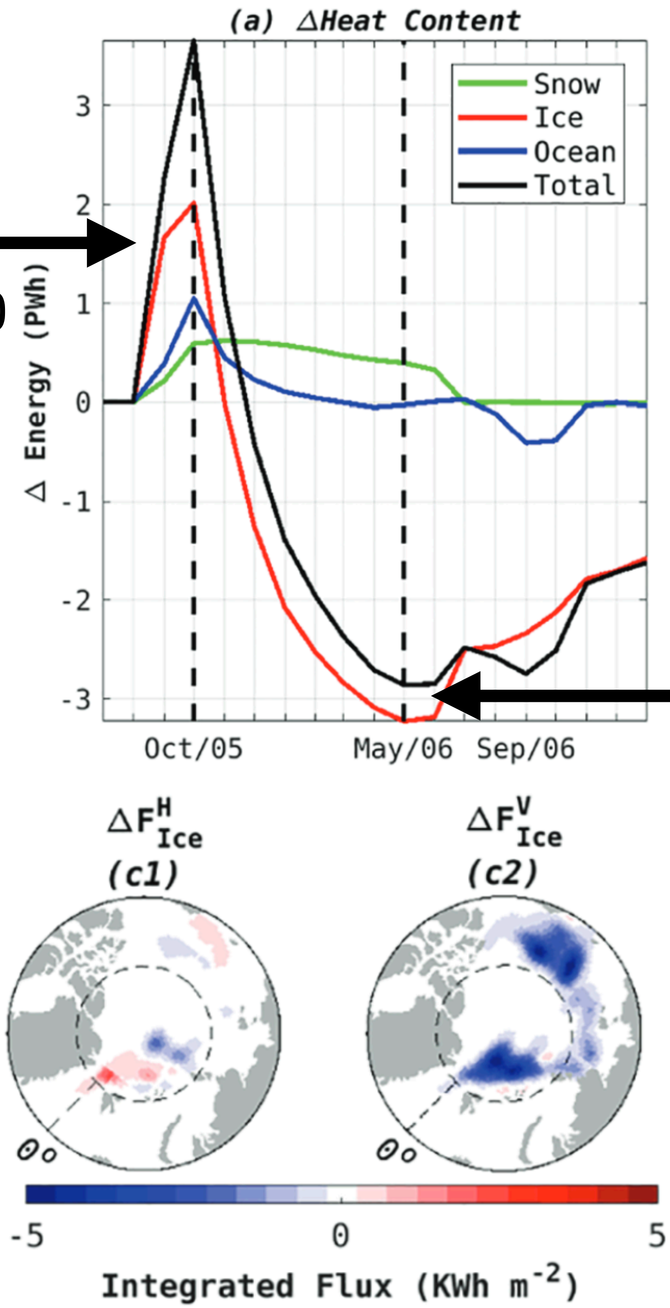


#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(c) System energy budget (WARM-CONTROL)

Energy gain due to applied T'>0 over ice



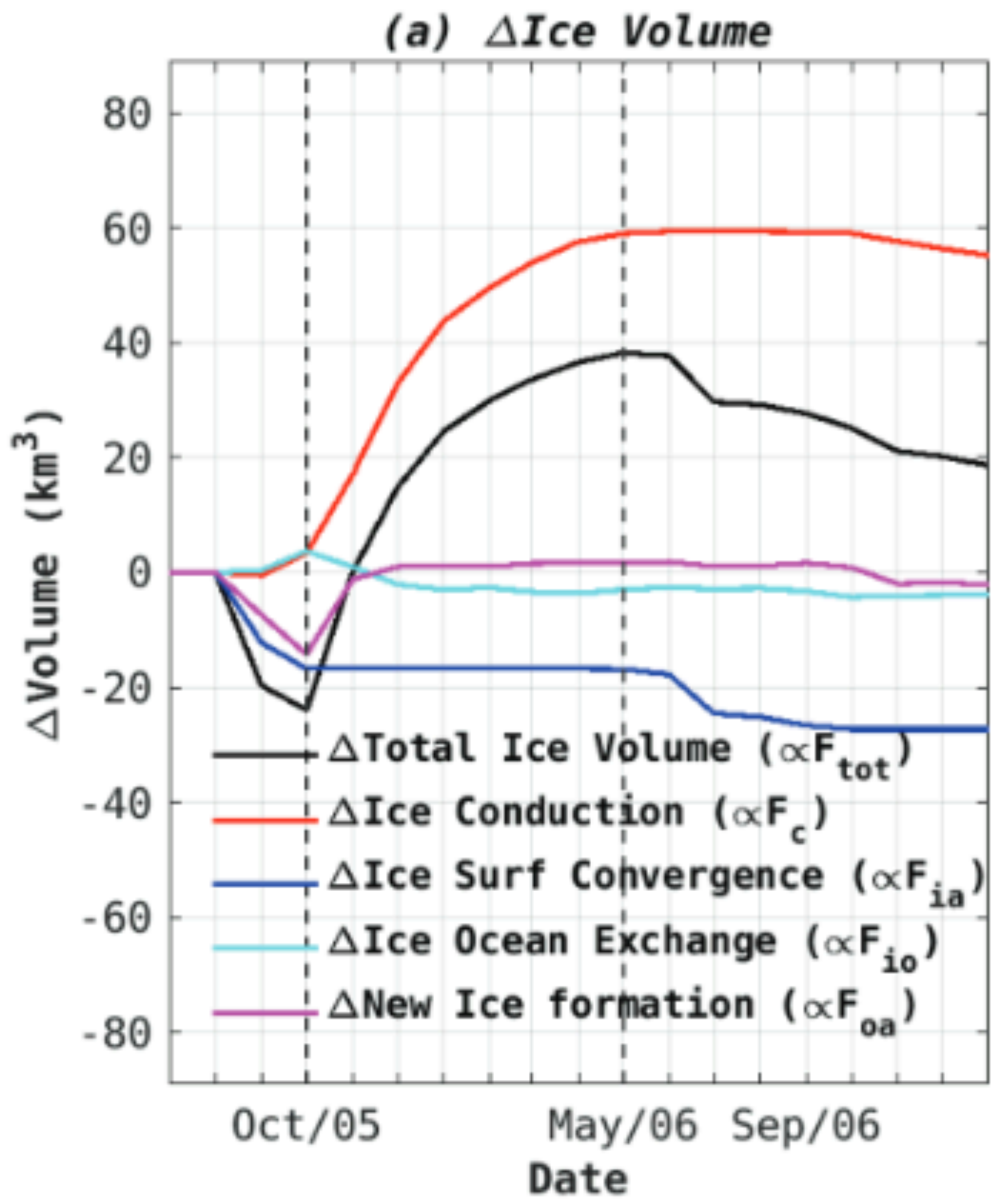
Subsequent energy loss >>0 initial energy gain

Due to increased vertical heat flux through ice

#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

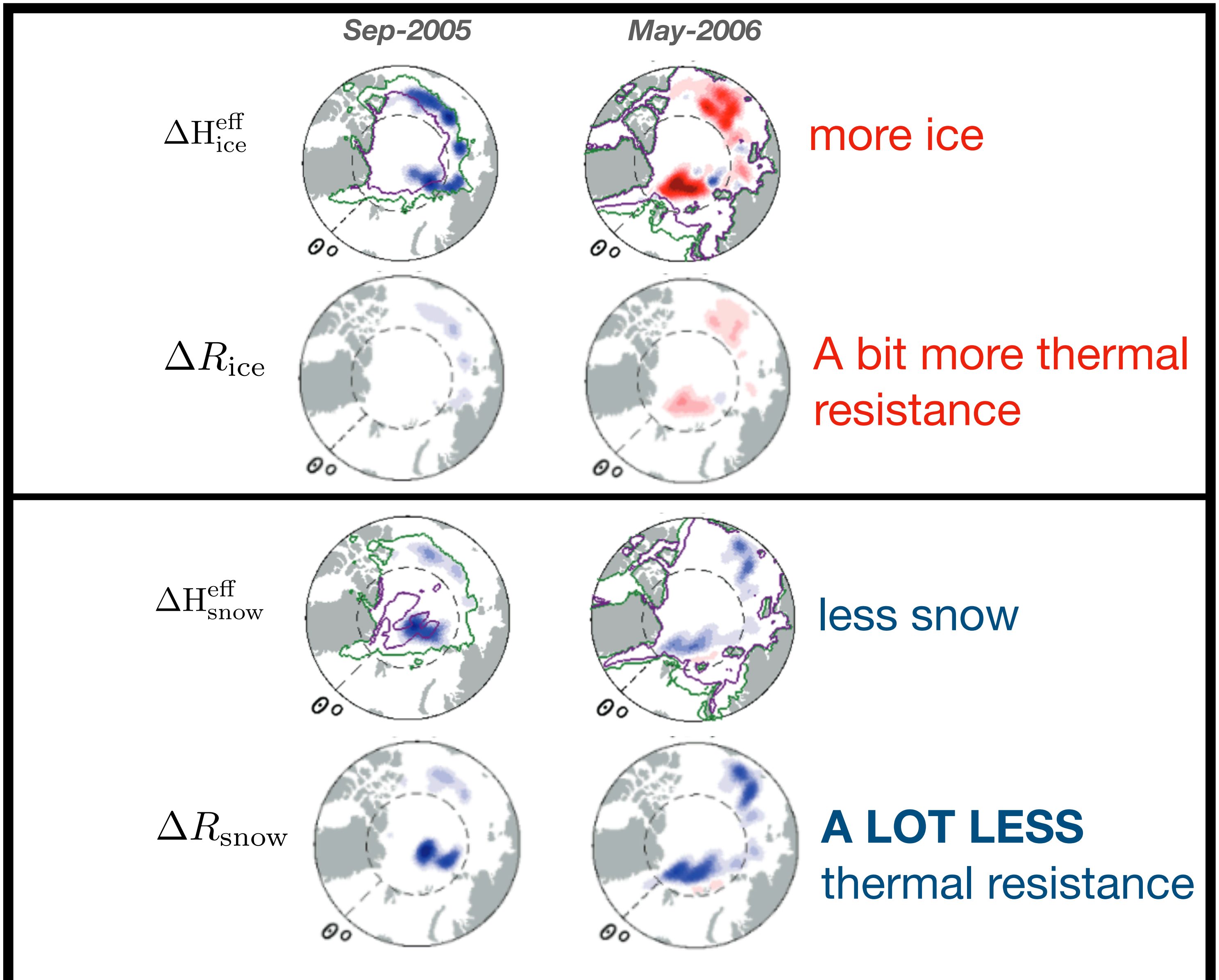
(d) Sea ice heat (equivalent ice volume) budget (WARM-CONTROL)



#3 Exploring drivers of Arctic sea ice variability

Bigdeli et al., 2020

(d) Δ thermal resistance (WARM-CONTROL) during period of significant sea ice loss in WARM



Snow resistance ~ 7 x ice resistance!

~35cm ice growth balances 5cm snow loss

#3 Exploring drivers of Arctic sea ice variability

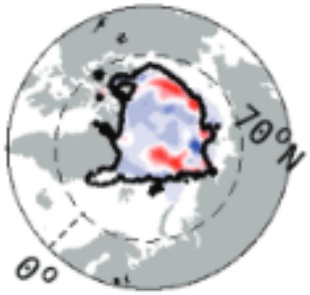
Bigdeli et al., 2020

Summary:

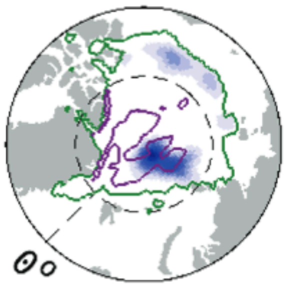
- Atmospheric warming can drive lagged sea ice growth via a **snow-melt-conductivity feedback**:
  - Atmospheric warming melts snow (+ ice)
  - **Loss of snow layer —> reduced thermal buffering —> increased conductive heat loss —> ice growth**
  - Increased thermal buffering from ice growth << reduced thermal buffering from snow loss —> ice growth continues; +ve anomaly still detectable at end of melt season
  - TIMING: snow melt most impactful at end of melt season; preconditions ice growth through winter

Complementarity of tools in ECCO

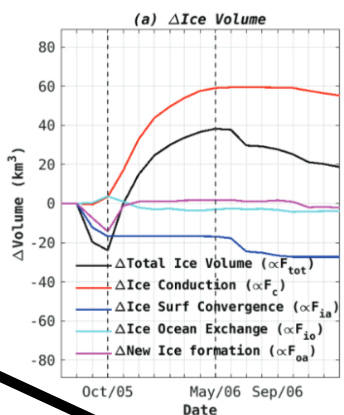
Adjoint sensitivity expriment



Forward perturbation experiments



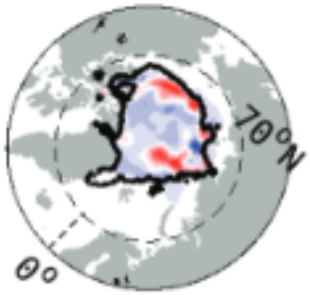
Budget assessment



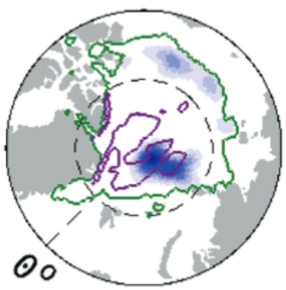
Comprehensive mechanistic explanation of local drivers of change

Complementarity of tools in ECCO

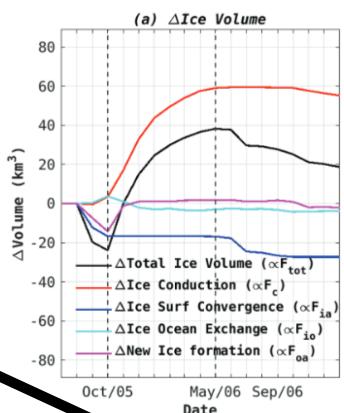
Adjoint sensitivity expriment

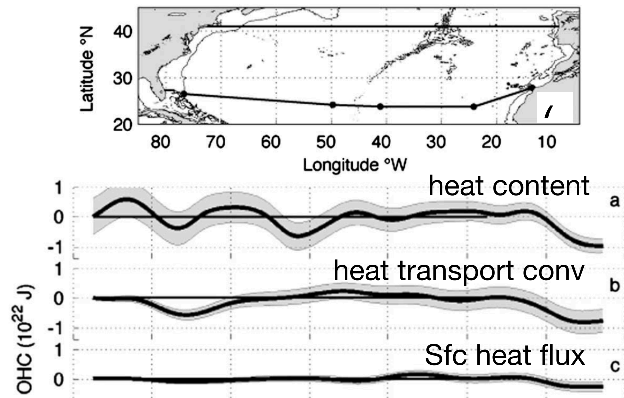


Forward perturbation experiments

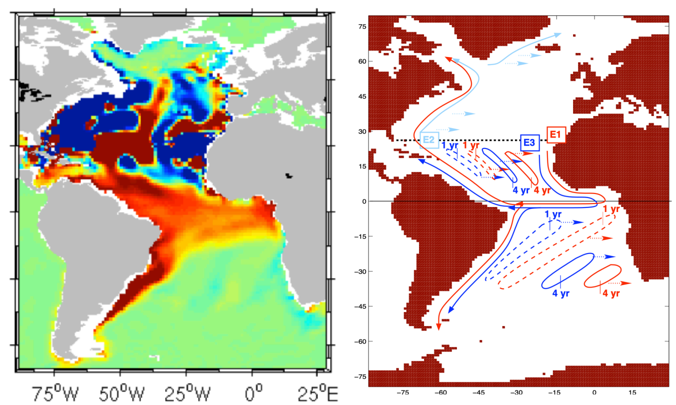


Budget assessment





Budget assessment



Adjoint sensitivity experiment

Comprehensive mechanistic explanation of local & remote drivers of change

SUMMARY

- Budgets analyses can provide a quantitative and mechanistic understanding ocean change

- Key advantages of ECCO include the ability to:
  1. (1) perform *meaningful* budget analyses in a data-constrained framework
  2. (2) link local changes to local & remote drivers (via budgets + adjoint sensitivities)

- Available ECCO diagnostics, documentation & tutorials can support your future analyses!

Thank You!

OCEAN BUDGETS IN ECCO
Part 2: Implementation

- Momentum
- Heat
- + hands on tutorial

Volume Budget Closure: See Ian & Andrew's tutorial in `ecco-2025` Jupyter book

ECCOv4 Global Volume Budget Closure

Here we demonstrate the closure of volume budgets in ECCOv4 configurations. This notebook is draws heavily from `evaluating_budgets_in_eccov4r3.pdf` which explains the procedure with Matlab code examples by Christopher G. Piecuch. See ECCO Version 4 release documents: */doc/evaluating_budgets_in_eccov4r3.pdf*).

Objectives

Illustrate how volume budgets are closed globally.

Introduction

ECCOv4 uses the z^* coordinate system in which the depth of the vertical coordinate, z^* varies with time as:

$$z^* = \frac{z - \eta(x,y,t)}{H(x,y) + \eta(x,y,t)} H(x,y)$$

With H being the model depth, η being the model sea level anomaly, and z being depth.

If the vertical coordinate didn't change through time then volume fluxes across the 'u' and 'v' grid cell faces of a tracer cell could be calculated by multiplying the velocities at the face with the face area:

volume flux across 'u' face in the +x direction =
$UVEL(x,y,k) \times drF(k) \times dyG(x,y) \times hFacW(x,y,k)$

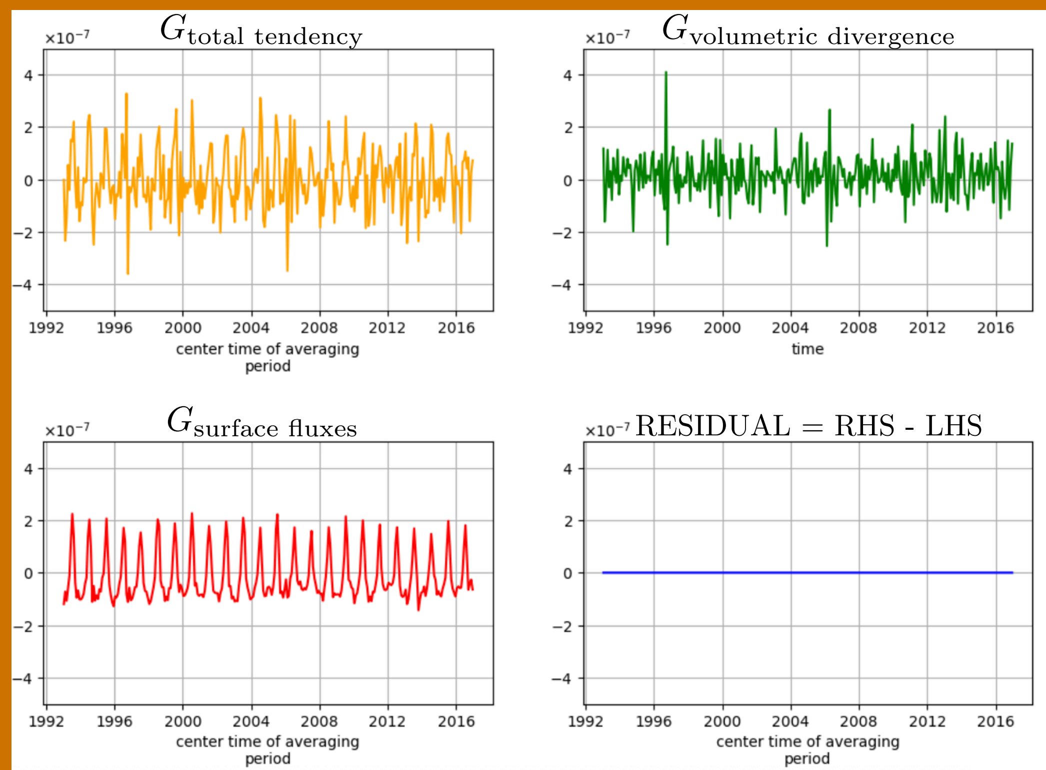
volume flux across 'v' face in the +y direction =
$VVEL(x,y,k) \times drF(k) \times dxG(x,y) \times hFacS(x,y,k)$

With `dyG` and `dxG` being the lengths of the 'u' and 'v' faces, `drF` being the grid cell height and `hFacW` and `hFacS` being the vertical fractions of the 'u' and 'v' grid cell faces that are open water (ECCOv4 uses partial cells to better represent bathymetry which can allows 0 < hfac $\leq$ 1).

However, because the vertical coordiate varies with time in the z^* system, the grid cell height `drF` varies with time as `drF` $\times s^*(t)$, with

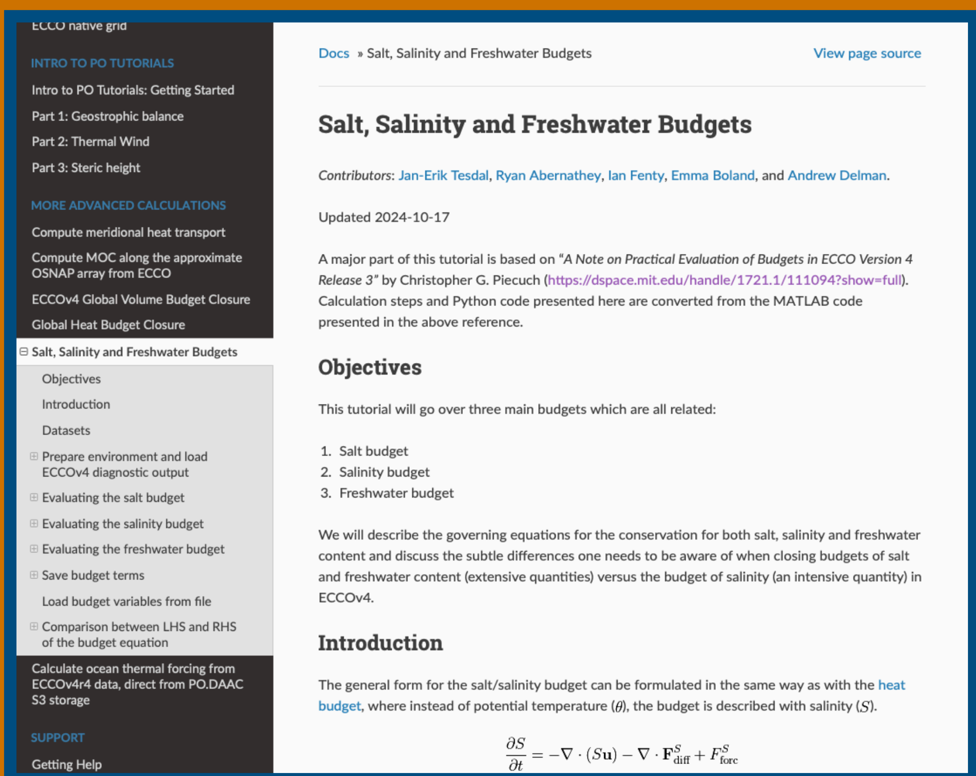
$$s^*(x,y,k,t) = 1 + \frac{\eta(x,y,t)}{H}$$

$$\underbrace{\frac{\partial \eta}{\partial t}}_{G_{\text{total tendency}}} = \underbrace{\int_{-H}^{0} \left(-\nabla_{z^*} (s^* \mathbf{v} - \frac{\partial w}{\partial z^*} \right) dz^*}_{G_{\text{volumetric divergence}}} + \underbrace{F}_{G_{surface fluxes}}$$



Salt/Salinity Budget Closure: See tutorial at https://ecco-v4-python-tutorial.readthedocs.io

$$\underbrace{\frac{\partial S}{\partial t}}_{G^{Sln}_{\text{total}}} = \underbrace{\frac{1}{s^*}\left[S\left(\nabla_{z^*}\cdot(s^*\mathbf{v})+\frac{\partial w}{\partial z^*}\right)-\nabla_{z^*}\cdot(s^* S\,\mathbf{v}_{res})-\frac{\partial(S\,w_{res})}{\partial z^*}\right]}_{G^{Sln}_{\text{advection}}} \underbrace{-\nabla\cdot\mathbf{F}^S_{\text{diff}}}_{G^{Sln}_{\text{diffusion}}} + \underbrace{F^S_{\text{forc}} - S\,\mathcal{F}}_{G^{Sln}_{\text{forcing}}}$$


ECCO native grid

INTRO TO PO TUTORIALS

Intro to PO Tutorials: Getting Started

Part 1: Geostrophic balance

Part 2: Thermal Wind

Part 3: Steric height

MORE ADVANCED CALCULATIONS

Compute meridional heat transport

Compute MOC along the approximate OSNAP array from ECCO

ECCOv4 Global Volume Budget Closure

Global Heat Budget Closure

Salt, Salinity and Freshwater Budgets

Objectives

Introduction

Datasets

Prepare environment and load ECCOv4 diagnostic output

Evaluating the salt budget

Evaluating the salinity budget

Evaluating the freshwater budget

Save budget terms

Load budget variables from file

Comparison between LHS and RHS of the budget equation

Calculate ocean thermal forcing from ECCOv4r4 data, direct from PO.DAAC S3 storage

SUPPORT

Getting Help

Docs » Salt, Salinity and Freshwater Budgets

View page source

Salt, Salinity and Freshwater Budgets

Contributors: Jan-Erik Tesdal, Ryan Abernathey, Ian Fenty, Emma Boland, and Andrew Delman.

Updated 2024-10-17

A major part of this tutorial is based on *"A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3"* by Christopher G. Piecuch (https://dspace.mit.edu/handle/1721.1/111094?show=full). Calculation steps and Python code presented here are converted from the MATLAB code presented in the above reference.

Objectives

This tutorial will go over three main budgets which are all related:

1. Salt budget
2. Salinity budget
3. Freshwater budget

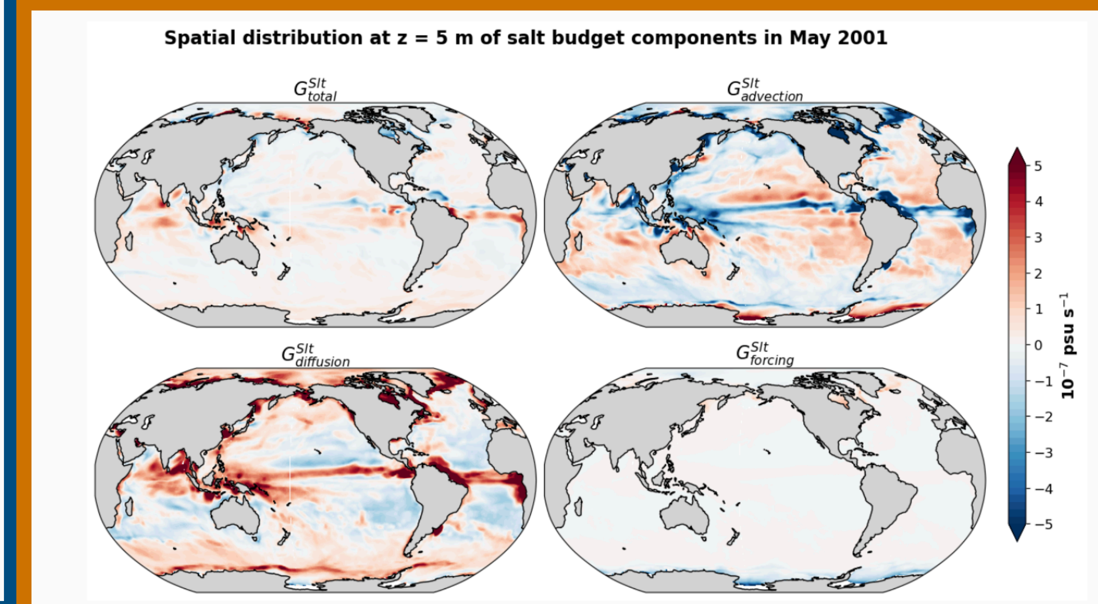
We will describe the governing equations for the conservation for both salt, salinity and freshwater content and discuss the subtle differences one needs to be aware of when closing budgets of salt and freshwater content (extensive quantities) versus the budget of salinity (an intensive quantity) in ECCOv4.

Introduction

The general form for the salt/salinity budget can be formulated in the same way as with the heat budget, where instead of potential temperature (θ), the budget is described with salinity (S).

$$\frac{\partial S}{\partial t} = -\nabla\cdot(S\mathbf{u}) - \nabla\cdot\mathbf{F}^S_{\text{diff}} + F^S_{\text{forc}}$$

$$\underbrace{\frac{\partial(s^* S)}{\partial t}}_{G^{Slt}_{\text{total}}} = \underbrace{-\nabla_{z^*}\cdot(s^* S\,\mathbf{v}_{res}) - \frac{\partial(S\,w_{res})}{\partial z^*}}_{G^{Slt}_{\text{advection}}} \underbrace{-s^*(\nabla\cdot\mathbf{F}^S_{\text{diff}})}_{G^{Slt}_{\text{diffusion}}} + \underbrace{s^* F^S_{\text{forc}}}_{G^{Slt}_{\text{forcing}}}$$



And Chris Piecuch's documentation & talk from ECCO 2019 Summer School

A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3

Christopher G. Piecuch (cpiecuch@aer.com)

June 16, 2017

Contents

1 Introduction **2**
1.1 Scope 2
2 Obtaining ECCOv4 Release 3 **3**
3 Model Configuration **3**
4 Budgets Evaluated for Extensive Quantities **5**
4.1 Volume Conservation 5
4.2 Heat Conservation 7
4.2.1 Geothermal Flux 10
4.3 Salt Conservation 12
5 Budgets Estimated for Intensive Quantities **16**
5.1 Salinity Budget 16
A Example Matlab Code and the gcmfaces Framework **18**
A.1 The gcmfaces Framework 18
A.2 Example Matlab code 19

1/15

Tracer Budgets in ECCO—Part II: Technical Details and Some Examples

Christopher G. Piecuch (cpiecuch@whoi.edu)
Woods Hole Oceanographic Institution, Woods Hole, MA

Estimating the Circulation and Climate of the Ocean (ECCO) Summer School
May 19-31, 2019 | Friday Harbor Laboratories | University of Washington

13/15

Diagnosing Approximate Budgets: Salinity

* use chain rule
* combine salt & volume conservation equations

$$\frac{\partial (s^* S)}{\partial t} = s^* \frac{\partial S}{\partial t} + S \frac{\partial s^*}{\partial t}$$

Proportional to tendency in salt cons. eq.

$$\frac{\partial S}{\partial t} = -\frac{1}{s^*}\left[S\frac{\partial s^*}{\partial t} + \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) + \frac{\partial\left(S w_{res}\right)}{\partial z^*}\right] + \mathcal{F}_S + D_S$$

$$\underbrace{\frac{\partial S}{\partial t}}_{G^{\dagger,tot}} = \underbrace{\frac{1}{s^*}\left[S\nabla_{z^*}\left(s^* \mathbf{v}\right) + S\frac{\partial w}{\partial z^*} - \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) - \frac{\partial\left(S w_{res}\right)}{\partial z^*}\right]}_{G^{\dagger,adv}} + \underbrace{\mathcal{F}_S - S\mathcal{F}}_{G^{\dagger,forc}} + \underbrace{D_S}_{G^{\dagger,diff}}$$

P-E+R

https://www.eccosummerschool.org/schedule-1

- These notes are for official ECCOV4r4 & 5
- Use diagnostics output on the native grid

ECCO Momentum Budgets

MITgcm Momentum Budget: equations

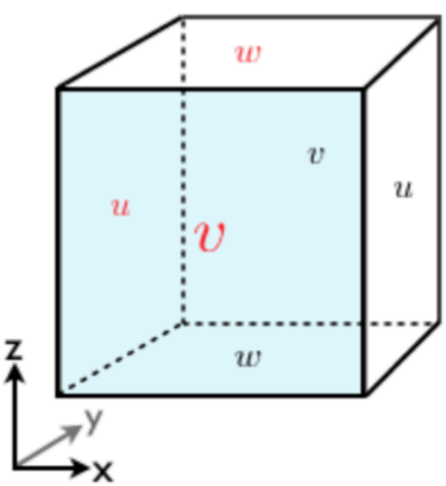
From JMC yesterday, vector invariant momentum eq:

$$\frac{\partial \mathbf{v}_h}{\partial t} = -(f+\zeta)\hat{\mathbf{k}} \times \mathbf{v}_h - \nabla \mathrm{KE} - w\frac{\partial \mathbf{v}_h}{\partial z} - g\nabla\eta - \frac{1}{\rho_c}\nabla_h p' + \nabla\cdot(\nu\nabla\mathbf{v}_h) + \frac{1}{\rho_c}\mathcal{F}_{\mathbf{v}}$$

Discretized u-mom:

$$G_u = G_u^{fv} + G_u^{\zeta_3 v} + G_u^{\zeta_2 w} + G_u^{\partial_x B} + G_u^{\partial_z \tau^x} + G_u^{h-\mathrm{diss}} + G_u^{v-\mathrm{diss}}$$

(a)



$$G_u^{fv} = \frac{1}{\Delta x_c}\overline{\frac{f}{h_\zeta}}^{j}\,\overline{\overline{\Delta x_g h_s v}^{j}}^{i}$$

$$\vdots$$

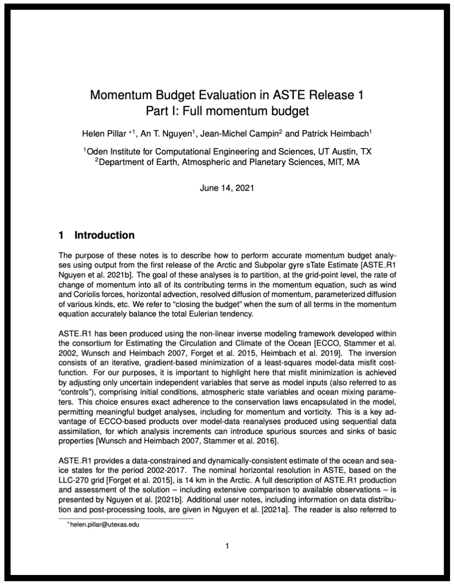
$$G_u^{v-\mathrm{diss}} = \frac{1}{\Delta r_f h_w}\delta_k \tau_{13}\,, \qquad \tau_{13} = A_v \frac{1}{\Delta r_c}\delta_k u$$

https://mitgcm.readthedocs.io

MITgcm Momentum Budget: annotated call tree

```
forward_step
|
|---dynamics
|     .
|     .
|     .
|    |CALL WRAPPER FOR MOMENTUM TENDENCY CALCULATIONS
|    |---mom_vecinv  if not using fluxform, call wrapper to compute tendencies in vector invariant eq.,
|    |     |         output = [guDissip,gvDissip] and net advective (inc. Coriolis) tendency [gU,gV] (via common block DYNVARS.h)
|     .
|     .
|     .
|    |     |
|    |     |---mom_calc_ke  KE at tracer point, needed for advective tendency, output=KE
|    |     |---mom_calc_relvort3  k̂·(∇×u), needed for Leith eddy viscosity, output=vort3, without dynamic BC applied
|    |     |---mom_calc_hdiv  (∇_h·u), needed for Leith eddy viscosity, output=hDiv
|    |     |
|    |     |
|    |     |BEFORE COMPUTING DISSIPATION, UPDATE BACKGROUND VISCOSITY TO INCLUDE EDDY VISCOSITY BASED ON LOCAL FLOW STRUCTURE
|    |     |IF (useVariableVisc) THEN  tension & strain also computed here but only relevant if using Smagorinsky closure
|    |     |                           (not in ASTE_R1)
|    |     |---mom_calc_visc  add to preset viscosity an eddy viscosity based on gradient of vorticity and divergence (Leith)
|    |     |                  If Smagorinsky is used, eddy viscosity is based on strain and tension instead
|    |     |                  output = updated harmonic and biharmonic coeffs. at corner and centre points. See MITgcm manual section 2.21.1
|    |     |ENDIF
|    |     |
|    |     |IF (useBiharmonicVisc)
|    |     |---mom_calc_del2uv  Harmonic parts (i.e., D, ζ terms, not D*, ζ* terms) of Eq. (12) output=[del2u,del2v]  (NB: ≠ ∇²u)
|    |     |---mom_calc_hdiv  2nd call computes ∇· [Eq. (12) without the D*, ζ* terms]. Gives ∇·(∇²u), output=dStar (Eq.(14))
|    |     |---mom_calc_relvort3  2nd call computes k̂·(∇× [Eq. (12) without the D*, ζ* terms]). Gives ∇²ζ_3, output=zStar (Eq.(15))
|    |     |ENDIF
|    |     |
|    |     |If (.NOT. useStrainTensionVisc) THEN  using vorticity and divergence formulation
|    |     |---mom_vi_hdissip  compute harmonic and biharmonic dissip. tendency as given by Eq. (12), from D, ζ, D*, ζ*
|    |     |      |          output = updated dissip. tendencies [guDiss,gvDiss], but still missing side and bottom drag
|    |     |      |          We have added a call to store the laplacian and biharmonic contributions separately
|    |     |      |CALL DIAGNOSTICS_FILL( uDiss_lap,'Um_Diss2',k,1,2,bi,bj,myThid)
|    |     |      |CALL DIAGNOSTICS_FILL( vDiss_lap,'Vm_Diss2',k,1,2,bi,bj,myThid)
|    |     |      |CALL DIAGNOSTICS_FILL( uDiss_bih,'Um_Diss4',k,1,2,bi,bj,myThid)
|    |     |      |CALL DIAGNOSTICS_FILL( vDiss_bih,'Vm_Diss4',k,1,2,bi,bj,myThid)
|    |     |      o
|    |     | ENDIF
|    |     |
|    |     | If (no_slip_sides) THEN
|    |     |---mom_u_sidedrag  u-mom tendency from no slip condition on viscous stress. Output = body force vF
|    |     |      |CALL DIAGNOSTICS_FILL( uDragTerms,'USidDrag',k,1,2,bi,bj,myThid)
|    |     |      o
|    |     | Do j=jMin,jMax  update u-mom diss. tendency after exiting side drag S/R
|    |     |  Do i=iMin,iMax
```

ASTE_R1
ECCOV4r4



Momentum Budget Evaluation in ASTE Release 1
Part I: Full momentum budget

Helen Pillar *[1], An T. Nguyen[1], Jean-Michel Campin[2] and Patrick Heimbach[1]

[1]Oden Institute for Computational Engineering and Sciences, UT Austin, TX
[2]Department of Earth, Atmospheric and Planetary Sciences, MIT, MA

June 14, 2021

1 Introduction

The purpose of these notes is to describe how to perform accurate momentum budget analyses using output from the first release of the Arctic and Subpolar gyre sTate Estimate [ASTE_R1 Nguyen et al. 2021b]. The goal of these analyses is to partition, at the grid-point level, the rate of change of momentum into all of its contributing terms in the momentum equation, such as wind and Coriolis forces, horizontal advection, resolved diffusion of momentum, parameterized diffusion of various kinds, etc. We refer to "closing the budget" when the sum of all terms in the momentum equation accurately balance the total Eulerian tendency.

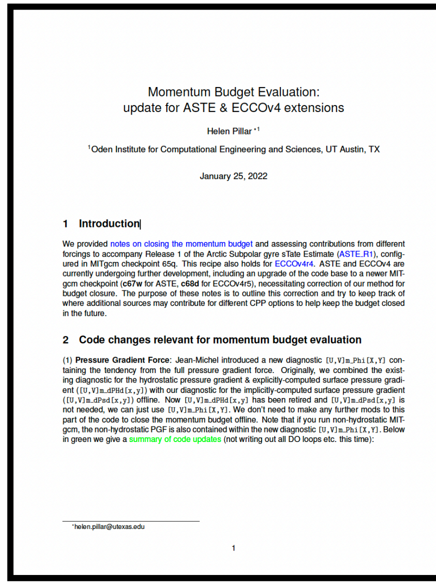
ASTE_R1 has been produced using the non-linear inverse modeling framework developed within the consortium for Estimating the Circulation and Climate of the Ocean [ECCO, Stammer et al. 2002, Wunsch and Heimbach 2007, Forget et al. 2015, Heimbach et al. 2019]. The inversion consists of an iterative, gradient-based minimization of a least-squares model-data misfit cost-function. For our purposes, it is important to highlight here that misfit minimization is achieved by adjusting only uncertain independent variables that serve as model inputs (also referred to as "controls"), comprising initial conditions, atmospheric state variables and ocean mixing parameters. This choice ensures exact adherence to the conservation laws encapsulated in the model, permitting meaningful budget analyses, including for momentum and vorticity. This is a key advantage of ECCO-based products over model-data reanalyses produced using sequential data assimilation, for which analysis increments can introduce spurious sources and sinks of basic properties [Wunsch and Heimbach 2007, Stammer et al. 2016].

ASTE_R1 provides a data-constrained and dynamically-consistent estimate of the ocean and sea-ice states for the period 2002-2017. The nominal horizontal resolution in ASTE, based on the LLC-270 grid [Forget et al. 2015], is 14 km in the Arctic. A full description of ASTE_R1 production and assessment of the solution – including extensive comparison to available observations – is presented by Nguyen et al. [2021b]. Additional user notes, including information on data distribution and post-processing tools, are given in Nguyen et al. [2021a]. The reader is also referred to

*helen.pillar@utexas.edu

1

ECCOV4r5



Momentum Budget Evaluation:
update for ASTE & ECCOv4 extensions

Helen Pillar *[1]

[1]Oden Institute for Computational Engineering and Sciences, UT Austin, TX

January 25, 2022

1 Introduction

We provided notes on closing the momentum budget and assessing contributions from different forcings to accompany Release 1 of the Arctic Subpolar gyre sTate Estimate (ASTE_R1), configured in MITgcm checkpoint 65q. This recipe also holds for ECCOv4r4. ASTE and ECCOv4 are currently undergoing further development, including an upgrade of the code base to a newer MITgcm checkpoint (**c67w** for ASTE, **c68d** for ECCOv4r5), necessitating correction of our method for budget closure. The purpose of these notes is to outline this correction and try to keep track of where additional sources may contribute for different CPP options to help keep the budget closed in the future.

2 Code changes relevant for momentum budget evaluation

(1) **Pressure Gradient Force**: Jean-Michel introduced a new diagnostic [U,V]m_Phi[X,Y] containing the tendency from the full pressure gradient force. Originally, we combined the existing diagnostic for the hydrostatic pressure gradient & explicitly-computed surface pressure gradient ([U,V]m_dPHd[x,y]) with our diagnostic for the implicitly-computed surface pressure gradient ([U,V]m_dPsd[x,y]) offline. Now [U,V]m_dPHd[x,y] has been retired and [U,V]m_dPsd[x,y] is not needed, we can just use [U,V]m_Phi[X,Y]. We don't need to make any further mods to this part of the code to close the momentum budget offline. Note that if you run non-hydrostatic MITgcm, the non-hydrostatic PGF is also contained within the new diagnostic [U,V]m_Phi[X,Y]. Below in green we give a summary of code updates (not writing out all DO loops etc. this time):

*helen.pillar@utexas.edu

1

MITgcm Momentum Budget: diagnostic output

Diagnostic	Description (units)	Units	Location	Dims
TOT[U,V]TEND	[u,v] total Eulerian tendency	$ms^{-1}day^{-1}$	[u,v]	[nx,ny,nz]
[U,V]m_Advec	[u,v] tendency from inertia + Coriolis	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_dPHd[X,Y]	[u,v] tendency from hydrostatic pressure gradient	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_dPsd[x,y]	[u,v] tendency from surface displacement	ms^{-2}	[u,v]	[nx,ny]
[U,V]m_Diss	[u,v] tendency from explicit dissipation	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_Impl	[u,v] tendency from implicit dissipation (vertical)	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_Ext	[u,v] tendency from external forcing (wind)	ms^{-2}	[u,v]	[nx,ny,nz]
AB_g[U,V]	[u,v] tendency from Adams-Bashforth timestepping	ms^{-2}	[u,v]	[nx,ny,nz]

$$\frac{\text{TOTUTEND(i,j,k)}}{86400} = \text{Um\_Advec(i,j,k)} + \text{Um\_dPHdx(i,j,k)} + \text{Um\_dPsdx(i,j)} \\ + \text{Um\_Diss(i,j,k)} + \text{Um\_Impl(i,j,k)} + \text{Um\_Ext(i,j,k)} + \text{AB\_gU(i,j,k)}$$

$$\frac{\text{TOTVTEND(i,j,k)}}{86400} = \text{Vm\_Advec(i,j,k)} + \text{Vm\_dPHdy(i,j,k)} + \text{Vm\_dPsdy(i,j)} \\ + \text{Vm\_Diss(i,j,k)} + \text{Vm\_Impl(i,j,k)} + \text{Vm\_Ext(i,j,k)} + \text{AB\_gV(i,j,k)}$$

for ASTE_R1 and ECCOV4r4. NB not true U/V, strictly X/Y-dir velocity

MITgcm Momentum Budget: explicit vs implicit

```
forward_step
|-- dynamics
      |-- calc_grad_phi_surf    Explicit sfc PGF, output = [phiSurfX, phiSurfY]
      do k = 1,Nr
      |-- calc_phi_hyd
            |-- calc_grad_phi_hyd    Hydrostatic PGF for level k, output = [dPhiHydX, dPhiHydY]
            o
      CALL DIAGNOSTIC_SCALE_FILL (dPhiHydX,tmpFac,1,'Um_dPHdx',k,1,2,bi,bj,myThid)
      |-- mom_vecinv    Compute remaining explicit tendencies (dissipation, Coriolis, inertia)

      |-- timestep    Compute remaining tendency from external forcing & accelerate flow
            |-- adams_bashforth2    Extrapolate forward
            o

      ENDDO
      o
|  Compute & apply forcing from implicit dissipation
|-- solve_for_pressure    Solve elliptic eq for p and update free surface displacement
|-- momentum_correction_step    velocity correction to remove divergence
      CALL DIAGNOSTIC_SCALE_FILL (gU_eta,'Um_dPsdx',1,1,2,bi,bj,myThid)
      o
o
```

e.g., PGF diagnostics in ASTE_R1

MITgcm Momentum Budget: diagnostic output

Diagnostic	Description (units)	Units	Location
TOT[U,V]TEND	[u,v] total Eulerian tendency	ms^{-2}	[u,v]
[U,V]m_Advec	[u,v] tendency from inertia + Coriolis	ms^{-2}	[u,v]
[U,V]m_dPhi[X,Y]	[u,v] tendency from pressure gradient (hydrostatic + surface)	ms^{-2}	[u,v]
[U,V]m_Diss	[u,v] tendency from explicit dissipation	ms^{-2}	[u,v]
[U,V]m_ImplD	[u,v] tendency from implicit dissipation (=vertical)	ms^{-2}	[u,v]
[U,V]m_Ext	[u,v] tendency from external forcing (=wind)	ms^{-2}	[u,v]
AB_g[U,V]	[u,v] tendency from Adams-Bashforth timestepping	ms^{-2}	[u,v]

$$\begin{aligned}\mathrm{TOTUTEND(i,j,k)} &= \mathrm{Um\_Advec(i,j,k)} + \mathrm{Um\_dPhiX(i,j,k)} \\ &+ \mathrm{Um\_Diss(i,j,k)} + \mathrm{Um\_ImplD(i,j,k)} + \mathrm{Um\_Ext(i,j,k)} + \mathrm{AB\_gU(i,j,k)}\end{aligned}$$

$$\begin{aligned}\mathrm{TOTVTEND(i,j,k)} &= \mathrm{Vm\_Advec(i,j,k)} + \mathrm{Vm\_dPhiY(i,j,k)} \\ &+ \mathrm{Vm\_Diss(i,j,k)} + \mathrm{Vm\_ImplD(i,j,k)} + \mathrm{Vm\_Ext(i,j,k)} + \mathrm{AB\_gV(i,j,k)}\end{aligned}$$

for ECCOV4r5. NB not true U/V, strictly X/Y-dir velocity on tiles

MITgcm Momentum Budget: diagnostic output

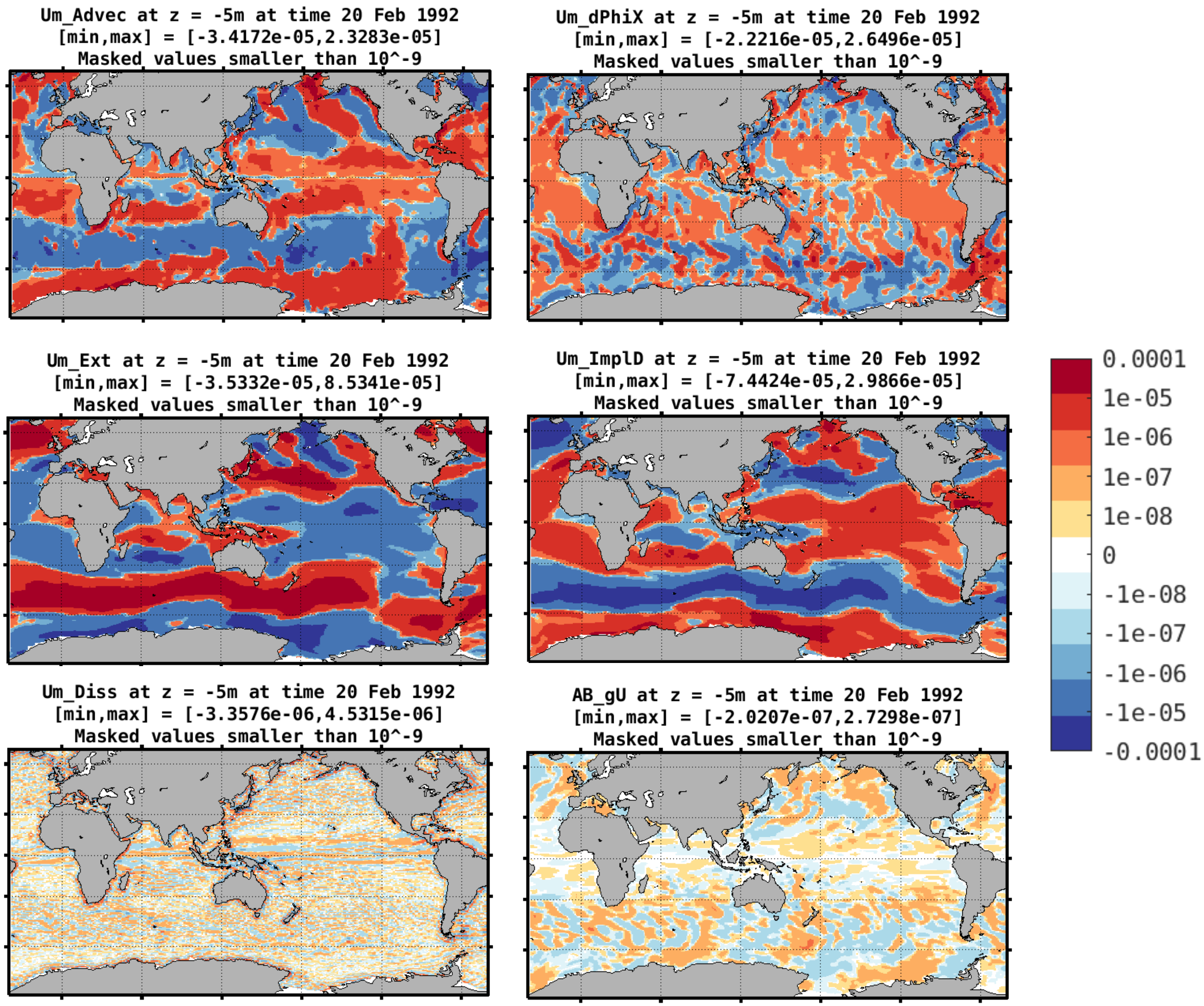
Diagnostic	Description (units)	Units	Location
TOT[U,V]TEND	[u,v] total Eulerian tendency	ms^{-2}	[u,v]
[U,V]m_Advec	[u,v] tendency from inertia + Coriolis	ms^{-2}	[u,v]
[U,V]m_dPhi[X,Y]	[u,v] tendency from pressure gradient (hydrostatic + surface)	ms^{-2}	[u,v]
[U,V]m_Diss	[u,v] tendency from explicit dissipation	ms^{-2}	[u,v]
[U,V]m_ImplD	[u,v] tendency from implicit dissipation (=vertical)	ms^{-2}	[u,v]
[U,V]m_Ext	[u,v] tendency from external forcing (=wind)	ms^{-2}	[u,v]
AB_g[U,V]	[u,v] tendency from Adams-Bashforth timestepping	ms^{-2}	[u,v]

To avoid distraction sometimes helps to:

- combine TOT[U,V]TEND and AB_g[U,V]
- combine [U,V]m_ImplD and [U,V]m_Diss

for ECCOV4r5. NB not true U/V, strictly X/Y-dir velocity on tiles

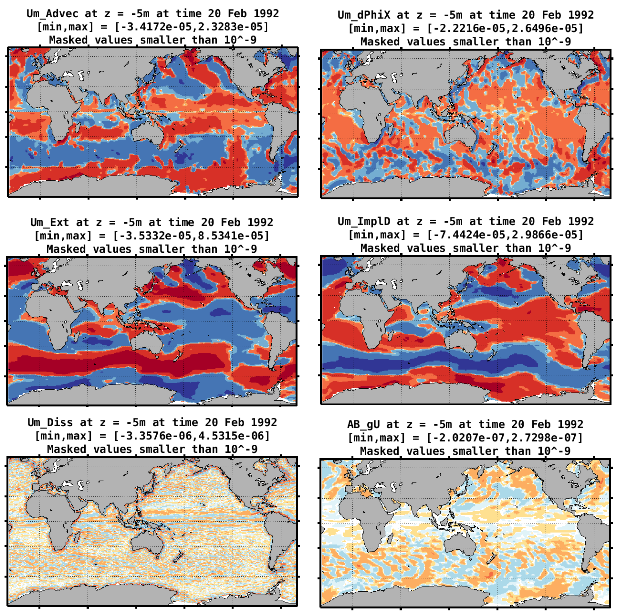
ECCOV4r5 Momentum Budget Closure



Example: ECCOV4r5 u-mom tendencies (RHS terms, ms^{-2}) at z=5m on 02/15/1992

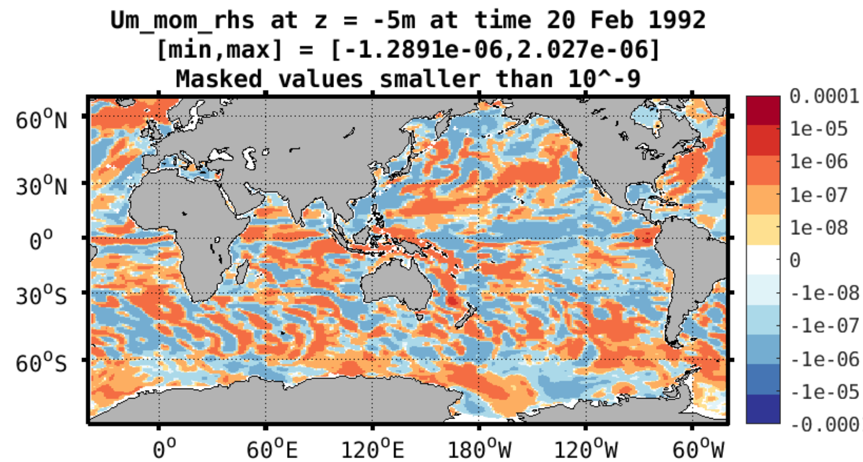
ECCCOV4r5 Momentum Budget Closure

$$\sum$$



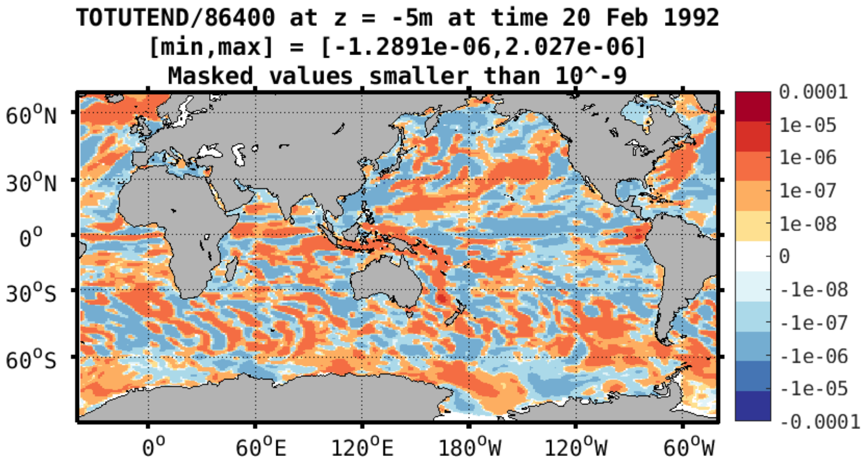
$$=$$

$\sum$ RHS terms



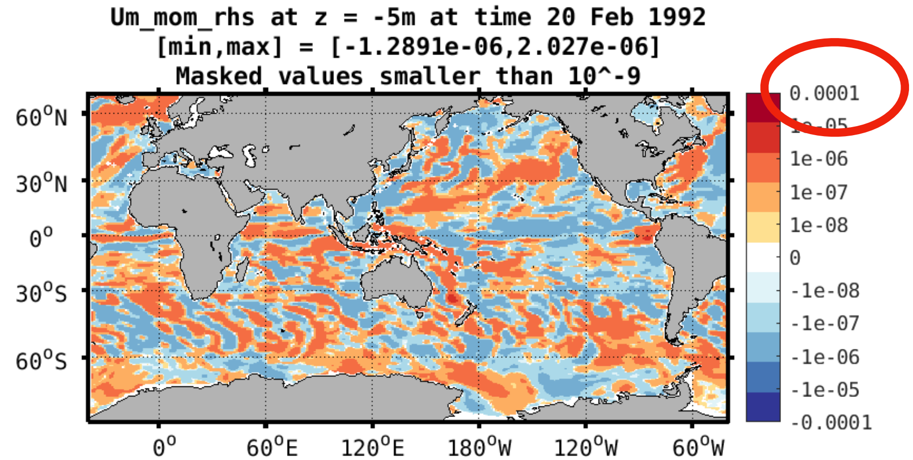
ECCCOV4r5 Momentum Budget Closure

TOTUTEND



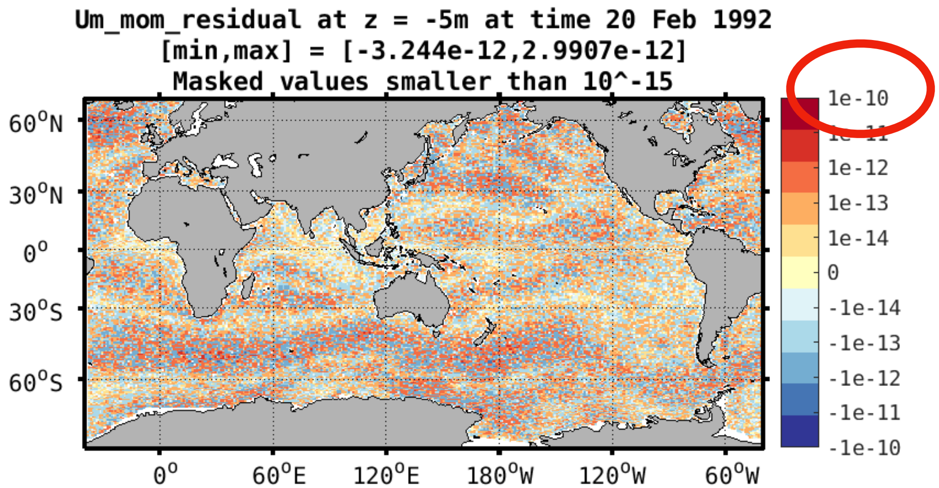
-

$\sum$ RHS terms



RESIDUAL

=



Momentum budget closure at every grid cell at all times

Momentum Budget Diagnostic Output - Extra terms

$$-\mathbf{u}\cdot\nabla\mathbf{u} = -\boldsymbol{\zeta}\times\mathbf{u} - \nabla\left[\frac{1}{2}(\mathbf{u}\cdot\mathbf{u})\right] = -\boldsymbol{\zeta}\times\mathbf{u} - \nabla KE,$$

$$= \begin{bmatrix} -\zeta_2 w + \zeta_3 v \\ \zeta_1 w - \zeta_3 u \\ -\zeta_1 v + \zeta_2 u \end{bmatrix} - \nabla KE,$$

Diagnostic	**Description (units)**	**Units**	**Location**	**Dims**
[U,V]m_Cori	[u,v] tendency from Coriolis	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_AdvRe	[u,v] tendency from vertical shears $[-\zeta_2 w, \zeta_1 w]$	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_AdvZ3	[u,v] tendency from "nonlinear Coriolis" $[\zeta_3 v, -\zeta_3 u]$	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_dKEd[x,y]	[u,v] tendency from KE gradient $[-\partial \mathrm{KE}/\partial x, -\partial \mathrm{KE}/\partial y]$	ms^{-2}	[u,v]	[nx,ny,nz]

$$\begin{aligned} \mathrm{Um\_Advec(i,j,k)} &= \mathrm{Um\_Cori(i,j,k)} \\ &\quad + \mathrm{Um\_AdvZ3(i,j,k)} + \mathrm{Um\_AdvRe(i,j,k)} + \mathrm{Um\_dKEdx(i,j,k)} \end{aligned}$$

(Analogous decomposition for Vm_Advec)

for ASTE_R1, ECCOV4r4, ECCOV4r5. NB not true U/V, strictly X/Y-dir velocity

Momentum Budget Diagnostic Output - Extra terms

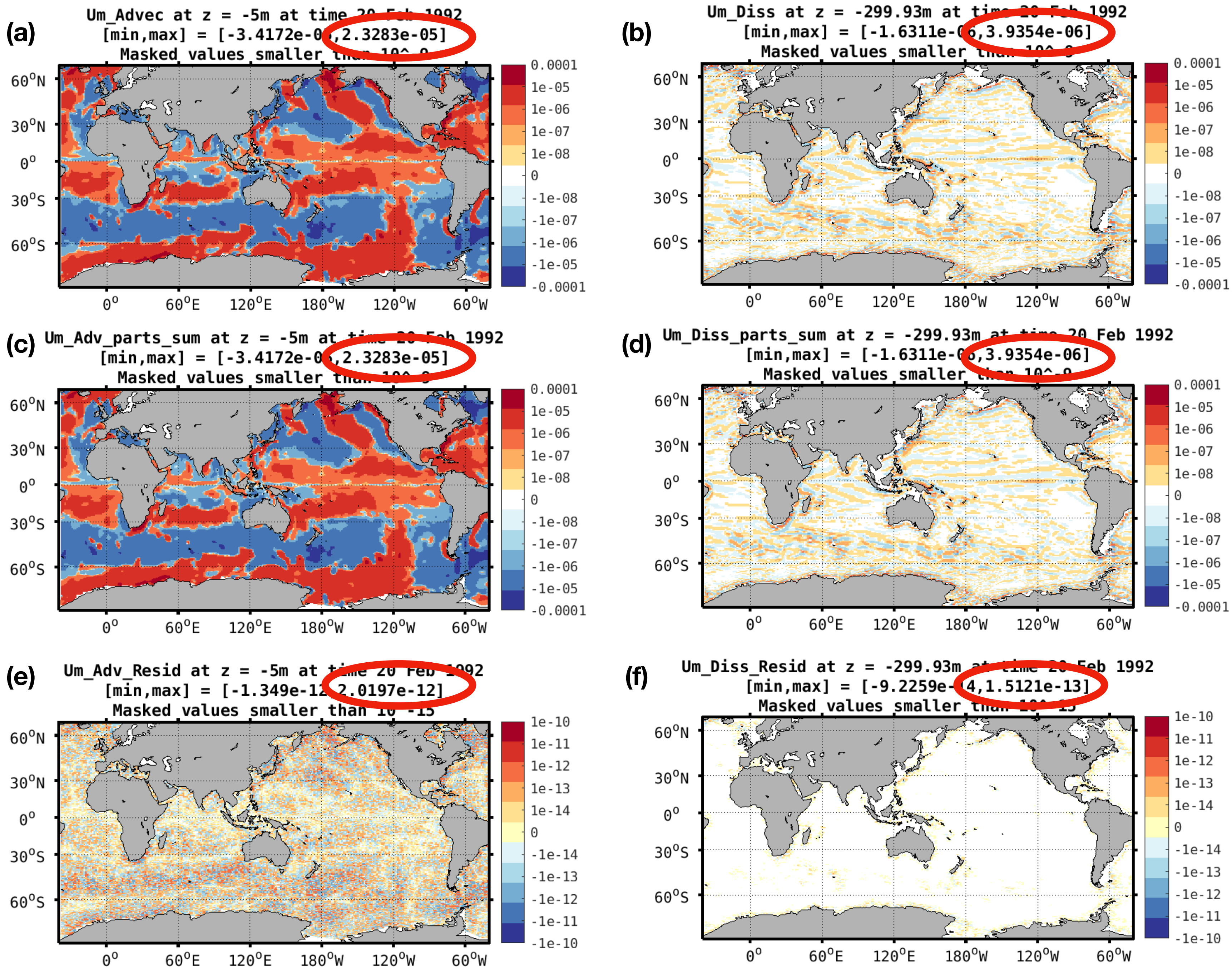
Diagnostic	Description (units)	Units	Location	Dims
[U,V]m_Diss2	[u,v] tendency from laplacian viscosity	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_Diss4	[u,v] tendency from biharmonic viscosity	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_SidDrag	[u,v] tendency from side drag	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_BotDrag	[u,v] tendency from bottom drag	ms^{-2}	[u,v]	[nx,ny,nz]
[U,V]m_ShIDrag	[u,v] tendency from shelf ice drag	ms^{-2}	[u,v]	[nx,ny,nz]

$$\begin{aligned}\mathrm{Um\_Diss(i,j,k)} = {} & \mathrm{Um\_Diss2(i,j,k)} + \mathrm{Um\_Diss4(i,j,k)} \\ & + \mathrm{UBotDrag(i,j,k)} + \mathrm{USidDrag(i,j,k)} + \mathrm{UShIDrag(i,j,k)}\end{aligned}$$

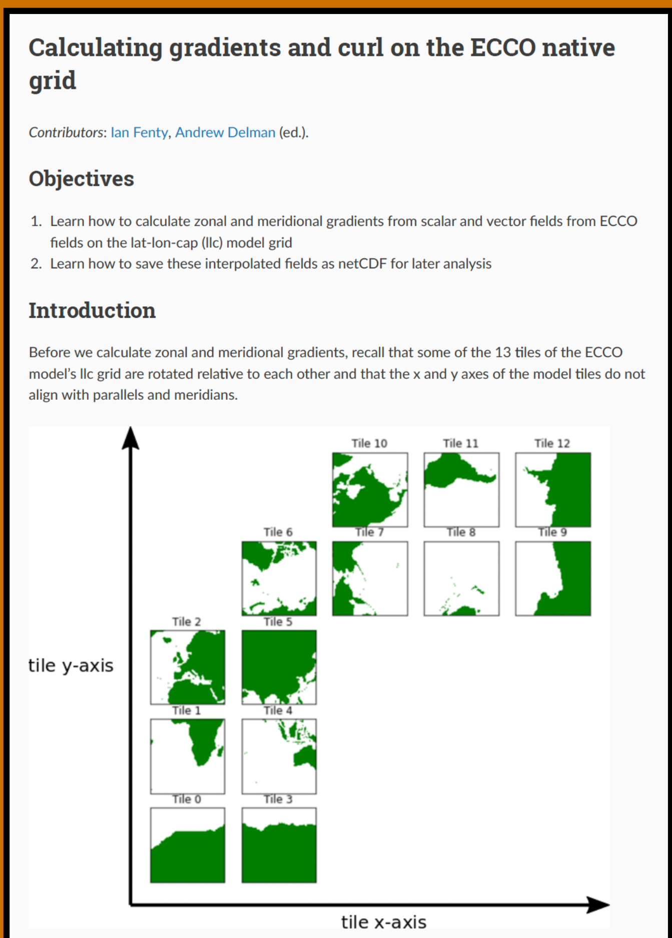
(Analogous decomposition for Vm_Diss)

for ASTE_R1, ECCOV4r4, ECCOV4r5. NB not true U/V, strictly X/Y-dir velocity

Momentum Budget Diagnostic Output - Extra terms



Vorticity Budgets: See Ian & Andrew's tutorial in `ecco-2025` Jupyter book



Calculating gradients and curl on the ECCO native grid

Contributors: Ian Fenty, Andrew Delman (ed.).

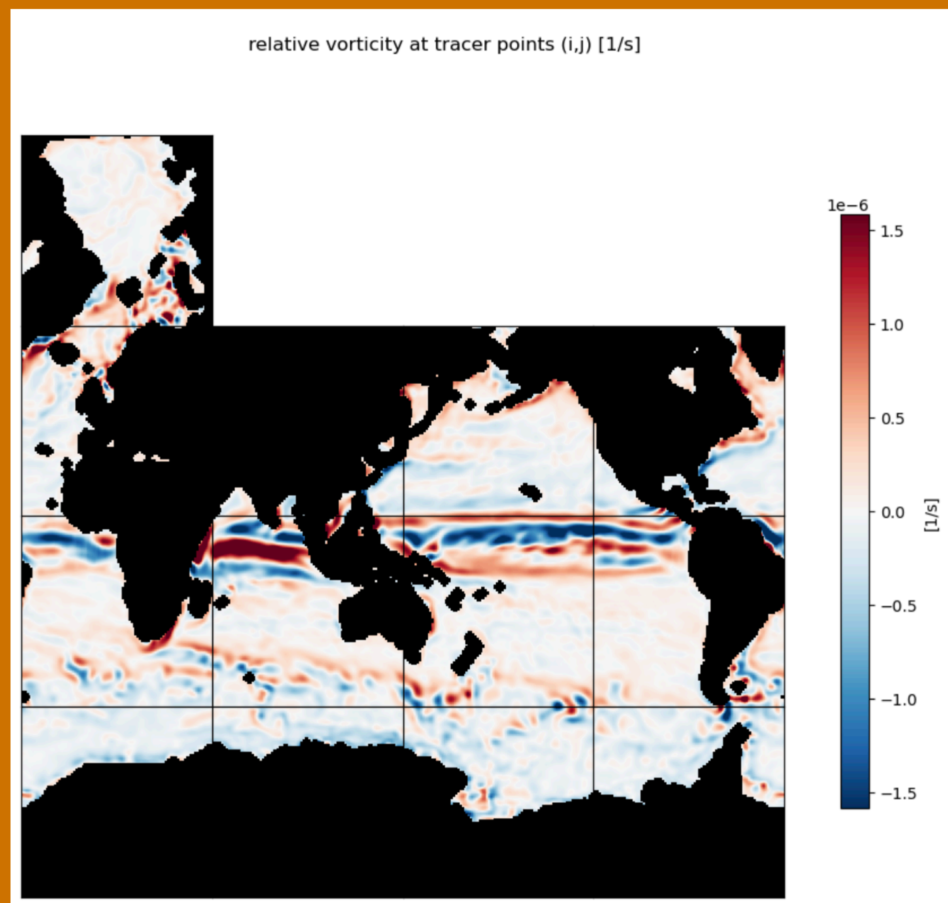
Objectives

1. Learn how to calculate zonal and meridional gradients from scalar and vector fields from ECCO fields on the lat-lon-cap (llc) model grid
2. Learn how to save these interpolated fields as netCDF for later analysis

Introduction

Before we calculate zonal and meridional gradients, recall that some of the 13 tiles of the ECCO model's llc grid are rotated relative to each other and that the x and y axes of the model tiles do not align with parallels and meridians.

Calculating gradients of vector fields located on the 'u' and 'v' points (vorticity/divergence)



ECCO Heat Budget Tutorial

MITgcm Heat Budget

$$\underbrace{\frac{\partial(s^*\theta)}{\partial t}}_{G^\theta_{\text{total tendency}}} = \underbrace{-\nabla_{z^*}(s^*\theta\mathbf{v}_{res}) - \frac{\partial(\theta w_{res})}{\partial z^*}}_{G^\theta_{\text{advection}}} - \underbrace{s^*(\nabla\cdot\mathbf{F}^\theta_{\text{diff}})}_{G^\theta_{\text{diffusion}}} + \underbrace{s^* F^\theta_{\text{sfc}}}_{G^\theta_{\text{forcing}}}$$

Rescaled height coordinate: $z^* = \dfrac{z-\eta(x,y,t)}{H(x,y)+\eta(x,y,t)}H(x,y)$

Scale factor: $s^*(x,y,k,t) = 1 + \dfrac{\eta(x,y,t)}{H(x,y)}$

MITgcm Heat Budget Diagnostics

Diagnostic	**Time**	**Description (units)**	**Units**
ETAN	Snapshot	Surface height anomaly	m
THETA	Snapshot	Potential temperature	°C
TFLUX	Average	Total heat flux	Wm^{-2}
oceQsw	Average	Net shortwave radiation	Wm^{-2}
ADVx_TH	Average	X-dir advective flux of pot. temp.	°C m^3s^{-1}
ADVy_TH	Average	Y-dir advective flux of pot. temp.	°C m^3s^{-1}
ADVr_TH	Average	Vertical advective flux of pot. temp.	°C m^3s^{-1}
DFrI_TH	Average	Implicit vertical diffusive flux of pot. temp.	°C m^3s^{-1}
DFrE_TH	Average	Explicit vertical diffusive flux of pot. temp.	°C m^3s^{-1}
DFxE_TH	Average	Explicit X-dir diffusive flux of pot. temp.	°C m^3s^{-1}
DFyE_TH	Average	Explicit Y-dir diffusive flux of pot. temp.	°C m^3s^{-1}

MITgcm Heat Tendency Terms

for $t = t_1, t_2, \ldots t_{T-1}, t_T$ **do** $\triangleright$ Loop over T time steps (months) t

$U_{i,j,k} = \texttt{ADVx\_TH}\{t\}$ $\triangleright$ 3-D **average** zonal advection over month t

$V_{i,j,k} = \texttt{ADVy\_TH}\{t\}$ $\triangleright$ 3-D **average** meridional advection over month t

$W_{i,j,k} = \texttt{ADVr\_TH}\{t\}$ $\triangleright$ 3-D **average** vertical advection over month t

$\mathcal{U}_{i,j,k} = \texttt{DFxE\_TH}\{t\}$ $\triangleright$ 3-D **average** zonal diffusion over month t

$\mathcal{V}_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ $\triangleright$ 3-D **average** meridional diffusion over month t

$\mathcal{W}^E_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ $\triangleright$ 3-D **average** vertical diffusion (explicit) over month t

$\mathcal{W}^I_{i,j,k} = \texttt{DFyI\_TH}\{t\}$ $\triangleright$ 3-D **average** vertical diffusion (implicit) over month t

$N^{(0)}_{i,j} = \texttt{ETAN}\{t - \Delta t\}$ $\triangleright$ 2-D surface height **snapshot** at start of month t

$N^{(f)}_{i,j} = \texttt{ETAN}\{t\}$ $\triangleright$ 2-D surface height **snapshot** at end of month t

$T^{(0)}_{i,j,k} = \texttt{THETA}\{t - \Delta t\}$ $\triangleright$ 3-D temperature **snapshot** at start of month t

$T^{(f)}_{i,j,k} = \texttt{THETA}\{t\}$ $\triangleright$ 3-D temperature **snapshot** at end of month t

$\upsilon_{i,j,k} = h_{i,j,k} A_{i,j} \Delta z_k$ $\triangleright$ Grid volume

for $i = i_1, i_2, \ldots i_{I-1}, i_I$ **do** $\triangleright$ Loop over I longitude cells i

for $j = j_1, j_2, \ldots j_{J-1}, j_J$ **do** $\triangleright$ Loop over J latitude cells j

$$s^{*(0)}_{i,j} = \left(1 + N^{(0)}_{i,j} / H_{i,j}\right)$$

$$s^{*(f)}_{i,j} = \left(1 + N^{(f)}_{i,j} / H_{i,j}\right)$$

for $k = k_1, k_2, \ldots k_{K-1}, k_K$ **do** $\triangleright$ Loop over K vertical cells k

$$G^{\theta,tot}_{i,j,k} = \left(T^{(f)}_{i,j,k} s^{*(f)}_{i,j} - T^{(0)}_{i,j,k} s^{*(0)}_{i,j}\right) / \Delta t$$

$$G^{\theta,advH}_{i,j,k} = \left(U_{i,j,k} - U_{i+1,j,k} + V_{i,j,k} - V_{i,j+1,k}\right) / \upsilon_{i,j,k}$$

$$G^{\theta,diffH}_{i,j,k} = \left(\mathcal{U}_{i,j,k} - \mathcal{U}_{i+1,j,k} + \mathcal{V}_{i,j,k} - \mathcal{V}_{i,j+1,k}\right) / \upsilon_{i,j,k}$$

$$G^{\theta,advV}_{i,j,k} = \left[\left(1 - \delta_{k,K}\right) W_{i,j,k+1} - W_{i,j,k}\right] / \upsilon_{i,j,k}$$

$$G^{\theta,diffV}_{i,j,k} = \left[\left(1 - \delta_{k,K}\right)\left(\mathcal{W}^E_{i,j,k+1} + \mathcal{W}^I_{i,j,k+1}\right) - \mathcal{W}^E_{i,j,k} - \mathcal{W}^I_{i,j,k}\right] / \upsilon_{i,j,k}$$

$$G^{\theta,adv}_{i,j,k} = G^{\theta,advH}_{i,j,k} + G^{\theta,advV}_{i,j,k}$$

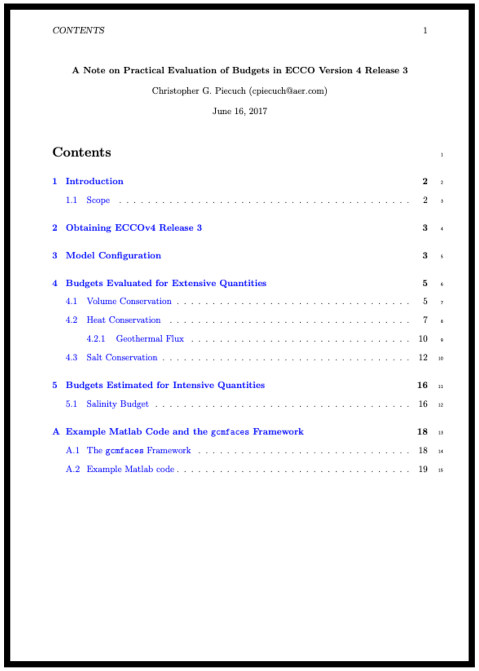
$$G^{\theta,diff}_{i,j,k} = G^{\theta,diffH}_{i,j,k} + G^{\theta,diffV}_{i,j,k}$$

end for

end for

end for

end for



CONTENTS 1

A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3

Christopher G. Piecuch (cpiecuch@aer.com)

June 16, 2017

Contents

1 Introduction 2
1.1 Scope 2
2 Obtaining ECCOv4 Release 3 3
3 Model Configuration 3
4 Budgets Evaluated for Extensive Quantities 5
4.1 Volume Conservation 5
4.2 Heat Conservation 7
4.2.1 Geothermal Flux 10
4.3 Salt Conservation 12
5 Budgets Estimated for Intensive Quantities 16
5.1 Salinity Budget 16
A Example Matlab Code and the gcmfaces Framework 18
A.1 The gcmfaces Framework 18
A.2 Example Matlab code 19

MITgcm Heat Tendency Terms

for $t = t_1, t_2, \ldots t_{T-1}, t_T$ **do** ▷ Loop over T time steps (months) t

$U_{i,j,k} = \texttt{ADVx\_TH}\{t\}$ ▷ 3-D **average** zonal advection over month t

$V_{i,j,k} = \texttt{ADVy\_TH}\{t\}$ ▷ 3-D **average** meridional advection over month t

$W_{i,j,k} = \texttt{ADVr\_TH}\{t\}$ ▷ 3-D **average** vertical advection over month t

$\mathcal{U}_{i,j,k} = \texttt{DFxE\_TH}\{t\}$ ▷ 3-D **average** zonal diffusion over month t

$\mathcal{V}_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ ▷ 3-D **average** meridional diffusion over month t

$\mathcal{W}^{E}_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ ▷ 3-D **average** vertical diffusion (explicit) over month t

$\mathcal{W}^{I}_{i,j,k} = \texttt{DFyI\_TH}\{t\}$ ▷ 3-D **average** vertical diffusion (implicit) over month t

$N^{(0)}_{i,j} = \texttt{ETAN}\{t - \Delta t\}$ ▷ 2-D surface height **snapshot** at start of month t

$N^{(f)}_{i,j} = \texttt{ETAN}\{t\}$ ▷ 2-D surface height **snapshot** at end of month t

$T^{(0)}_{i,j,k} = \texttt{THETA}\{t - \Delta t\}$ ▷ 3-D temperature **snapshot** at start of month t

$T^{(f)}_{i,j,k} = \texttt{THETA}\{t\}$ ▷ 3-D temperature **snapshot** at end of month t

$v_{i,j,k} = h_{i,j,k} A_{i,j} \Delta z_k$ ▷ Grid volume

for $i = i_1, i_2, \ldots i_{I-1}, i_I$ **do** ▷ Loop over I longitude cells i

for $j = j_1, j_2, \ldots j_{J-1}, j_J$ **do** ▷ Loop over J latitude cells j

$s^{*(0)}_{i,j} = \left(1 + N^{(0)}_{i,j} / H_{i,j}\right)$

$s^{*(f)}_{i,j} = \left(1 + N^{(f)}_{i,j} / H_{i,j}\right)$

for $k = k_1, k_2, \ldots k_{K-1}, k_K$ **do** ▷ Loop over K vertical cells k

$\boxed{G^{\theta,tot}_{i,j,k} = \left(T^{(f)}_{i,j,k} s^{*(f)}_{i,j} - T^{(0)}_{i,j,k} s^{*(0)}_{i,j}\right) / \Delta t}$

$G^{\theta,advH}_{i,j,k} = \left(U_{i,j,k} - U_{i+1,j,k} + V_{i,j,k} - V_{i,j+1,k}\right) / v_{i,j,k}$

$G^{\theta,diffH}_{i,j,k} = \left(\mathcal{U}_{i,j,k} - \mathcal{U}_{i+1,j,k} + \mathcal{V}_{i,j,k} - \mathcal{V}_{i,j+1,k}\right) / v_{i,j,k}$

$G^{\theta,advV}_{i,j,k} = \left[\left(1 - \delta_{k,K}\right) W_{i,j,k+1} - W_{i,j,k}\right] / v_{i,j,k}$

$G^{\theta,diffV}_{i,j,k} = \left[\left(1 - \delta_{k,K}\right)\left(\mathcal{W}^{E}_{i,j,k+1} + \mathcal{W}^{I}_{i,j,k+1}\right) - \mathcal{W}^{E}_{i,j,k} - \mathcal{W}^{I}_{i,j,k}\right] / v_{i,j,k}$

$G^{\theta,adv}_{i,j,k} = G^{\theta,advH}_{i,j,k} + G^{\theta,advV}_{i,j,k}$

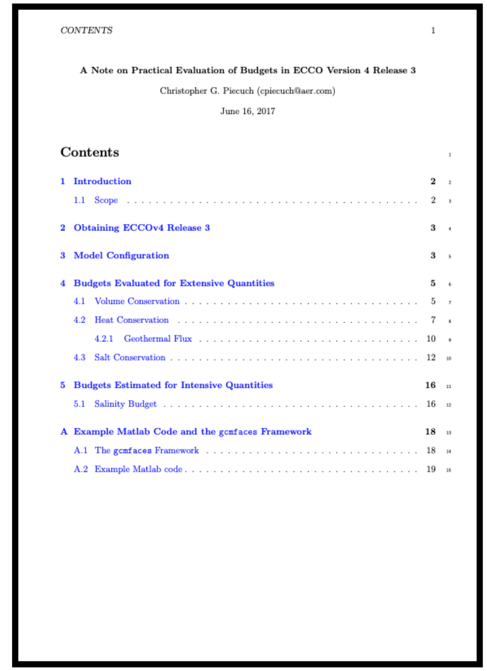
$G^{\theta,diff}_{i,j,k} = G^{\theta,diffH}_{i,j,k} + G^{\theta,diffV}_{i,j,k}$

end for

end for

end for

end for



CONTENTS 1

A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3

Christopher G. Piecuch (cpiecuch@aer.com)

June 16, 2017

Contents

1 Introduction 2

1.1 Scope 2

2 Obtaining ECCOv4 Release 3 3

3 Model Configuration 3

4 Budgets Evaluated for Extensive Quantities 5

4.1 Volume Conservation 5

4.2 Heat Conservation 7

4.2.1 Geothermal Flux 10

4.3 Salt Conservation 12

5 Budgets Estimated for Intensive Quantities 16

5.1 Salinity Budget 16

A Example Matlab Code and the gcmfaces Framework 18

A.1 The gcmfaces Framework 18

A.2 Example Matlab code 19

snapshot files: $\${\texttt{varname}}.\${\texttt{ITER}_{t2}}\texttt{.data}$ $\${\texttt{varname}}.\${\texttt{ITER}_{t3}}\texttt{.data}$

monthly mean files written at end of averaging period: $\${\texttt{varname}}.\${\texttt{ITER}_{t3}}\texttt{.data}$

MITgcm Heat Tendency Terms

for $t = t_1, t_2, \dots t_{T-1}, t_T$ **do** ▷ Loop over T time steps (months) t

$\boxed{U_{i,j,k} = \texttt{ADVx\_TH}\{t\}}$ ▷ 3-D **average** zonal advection over month t

$\boxed{V_{i,j,k} = \texttt{ADVy\_TH}\{t\}}$ ▷ 3-D **average** meridional advection over month t

$W_{i,j,k} = \texttt{ADVr\_TH}\{t\}$ ▷ 3-D **average** vertical advection over month t

$\mathcal{U}_{i,j,k} = \texttt{DFxE\_TH}\{t\}$ ▷ 3-D **average** zonal diffusion over month t

$\mathcal{V}_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ ▷ 3-D **average** meridional diffusion over month t

$\mathcal{W}^{E}_{i,j,k} = \texttt{DFyE\_TH}\{t\}$ ▷ 3-D **average** vertical diffusion (explicit) over month t

$\mathcal{W}^{I}_{i,j,k} = \texttt{DFyI\_TH}\{t\}$ ▷ 3-D **average** vertical diffusion (implicit) over month t

$N^{(0)}_{i,j} = \texttt{ETAN}\{t - \Delta t\}$ ▷ 2-D surface height **snapshot** at start of month t

$N^{(f)}_{i,j} = \texttt{ETAN}\{t\}$ ▷ 2-D surface height **snapshot** at end of month t

$T^{(0)}_{i,j,k} = \texttt{THETA}\{t - \Delta t\}$ ▷ 3-D temperature **snapshot** at start of month t

$T^{(f)}_{i,j,k} = \texttt{THETA}\{t\}$ ▷ 3-D temperature **snapshot** at end of month t

$\boxed{v_{i,j,k} = h_{i,j,k} A_{i,j} \Delta z_k}$ ▷ Grid volume

for $i = i_1, i_2, \dots i_{I-1}, i_I$ **do** ▷ Loop over I longitude cells i

for $j = j_1, j_2, \dots j_{J-1}, j_J$ **do** ▷ Loop over J latitude cells j

$$s^{*(0)}_{i,j} = \left(1 + N^{(0)}_{i,j} / H_{i,j}\right)$$

$$s^{*(f)}_{i,j} = \left(1 + N^{(f)}_{i,j} / H_{i,j}\right)$$

for $k = k_1, k_2, \dots k_{K-1}, k_K$ **do** ▷ Loop over K vertical cells k

$$G^{\theta,tot}_{i,j,k} = \left(T^{(f)}_{i,j,k} s^{*(f)}_{i,j} - T^{(0)}_{i,j,k} s^{*(0)}_{i,j}\right) / \Delta t$$

$$\boxed{G^{\theta,advH}_{i,j,k} = \left(U_{i,j,k} - U_{i+1,j,k} + V_{i,j,k} - V_{i,j+1,k}\right) / v_{i,j,k}}$$

$$G^{\theta,diffH}_{i,j,k} = \left(\mathcal{U}_{i,j,k} - \mathcal{U}_{i+1,j,k} + \mathcal{V}_{i,j,k} - \mathcal{V}_{i,j+1,k}\right) / v_{i,j,k}$$

$$G^{\theta,advV}_{i,j,k} = \left[(1 - \delta_{k,K}) W_{i,j,k+1} - W_{i,j,k}\right] / v_{i,j,k}$$

$$G^{\theta,diffV}_{i,j,k} = \left[(1 - \delta_{k,K}) \left(\mathcal{W}^{E}_{i,j,k+1} + \mathcal{W}^{I}_{i,j,k+1}\right) - \mathcal{W}^{E}_{i,j,k} - \mathcal{W}^{I}_{i,j,k}\right] / v_{i,j,k}$$

$$G^{\theta,adv}_{i,j,k} = G^{\theta,advH}_{i,j,k} + G^{\theta,advV}_{i,j,k}$$

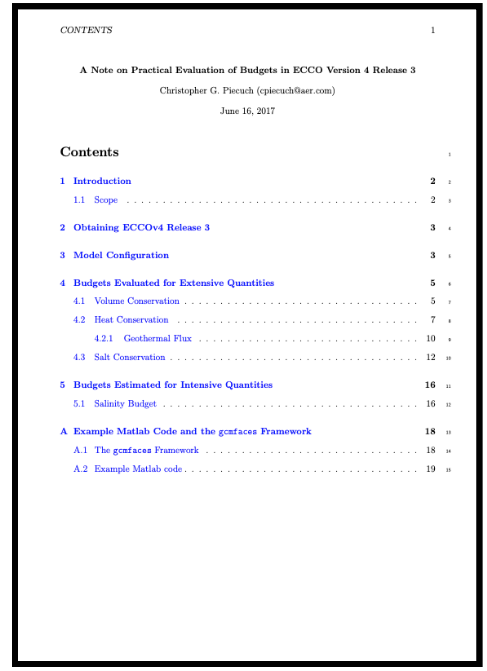
$$G^{\theta,diff}_{i,j,k} = G^{\theta,diffH}_{i,j,k} + G^{\theta,diffV}_{i,j,k}$$

end for

end for

end for

end for


CONTENTS 1

A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3

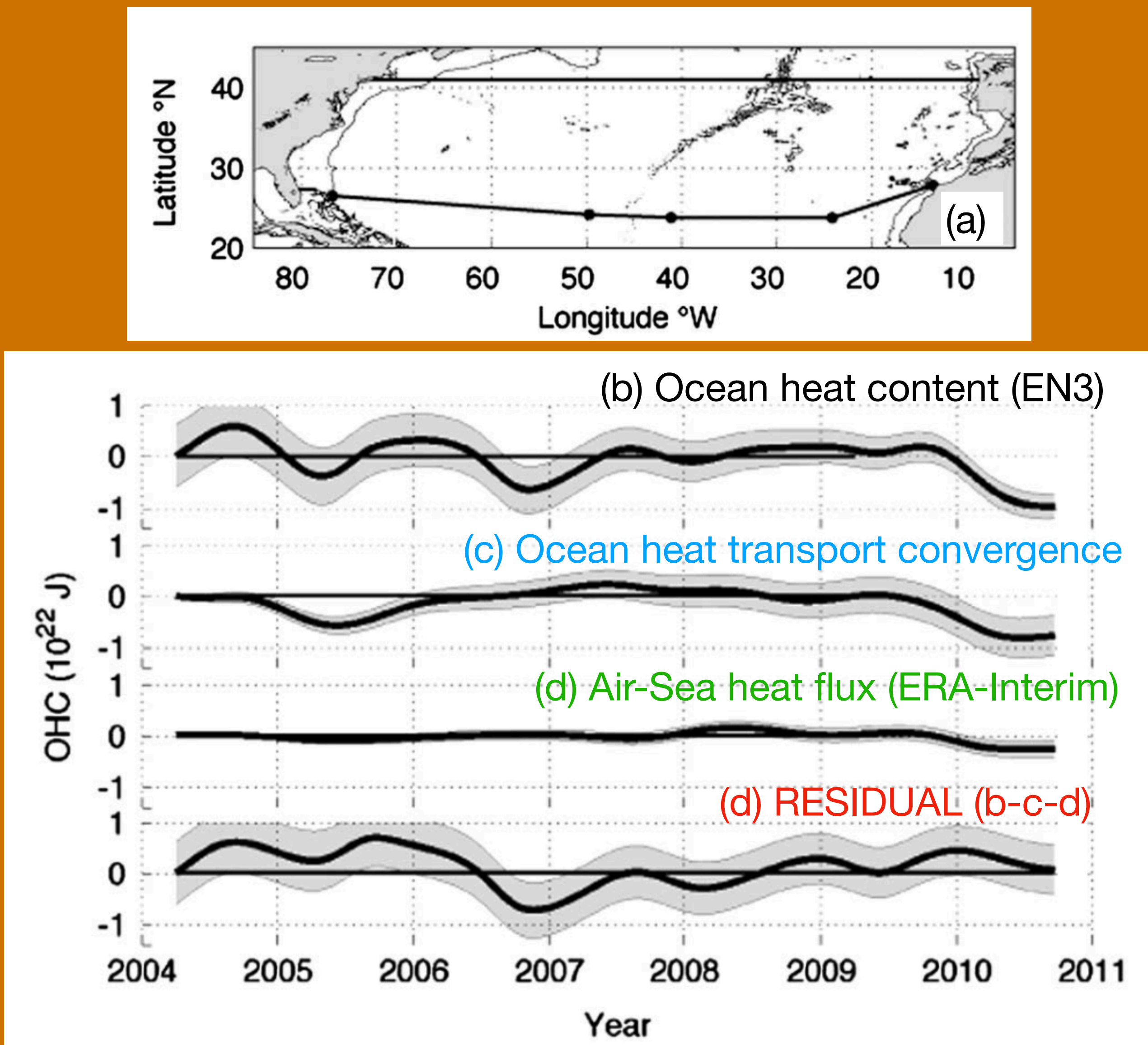
Christopher G. Piecuch (cpiecuch@aer.com)

June 16, 2017

Contents

1 Introduction 2
1.1 Scope 2
2 Obtaining ECCOv4 Release 3 3
3 Model Configuration 3
4 Budgets Evaluated for Extensive Quantities 5
4.1 Volume Conservation 5
4.2 Heat Conservation 7
4.2.1 Geothermal Flux 10
4.3 Salt Conservation 12
5 Budgets Estimated for Intensive Quantities 16
5.1 Salinity Budget 16
A Example Matlab Code and the gcmfaces Framework 18
A.1 The gcmfaces Framework 18
A.2 Example Matlab code 19

Revisit Cunningham et al., 2014 heat budget calculation in the North Atlantic



<u>Objectives</u>

- Diagnose V4r5 OHC’
- Obtain closed budget
- Determine driver of cold anomaly