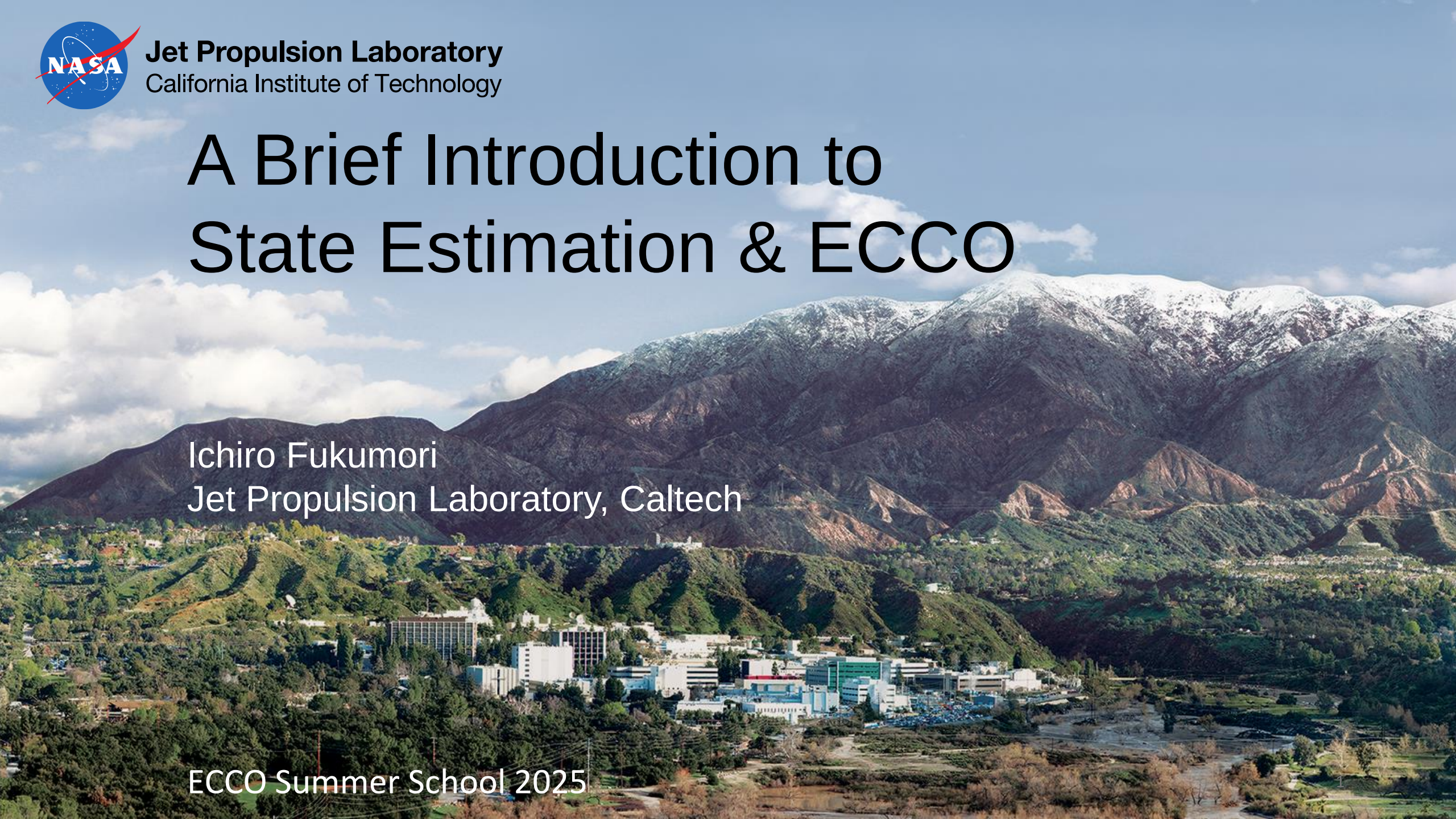


Jet Propulsion Laboratory
California Institute of Technology

A Brief Introduction to State Estimation & ECCO

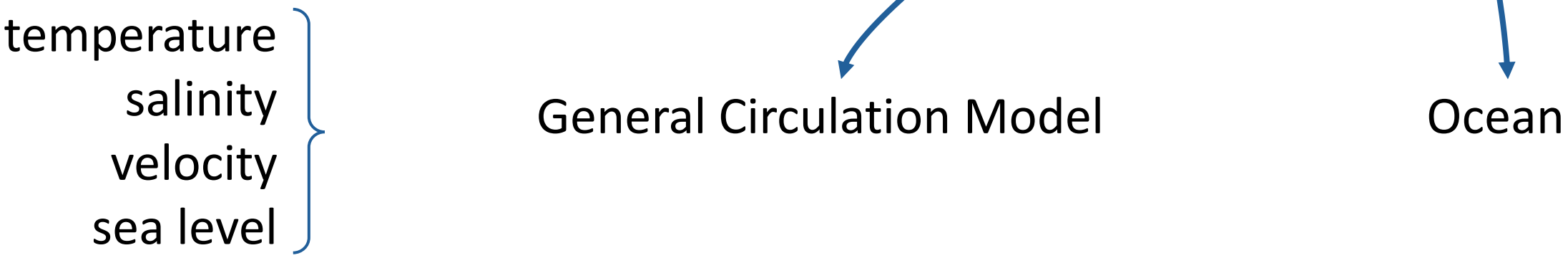
Ichiro Fukumori
Jet Propulsion Laboratory, Caltech

ECCO Summer School 2025

What is State Estimation?

"State Estimation" is data analysis using models.

A process to estimate the state of a dynamic system
by combining observations with a theoretical model
in a mathematically principled framework.



Related to "Data Assimilation."

Data Analysis Using Models (1/2)

"Geostrophic Calculation" aka "Dynamic Computation"

Sandstrom, J.W., and B. Helland-Hansen (1903), "Uber die Berechnung von Meeresstromungen," Report on Norwegian Fishery and Marine Investigations 2 (No. 4).

Geostrophic balance

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

Hydrostatic balance

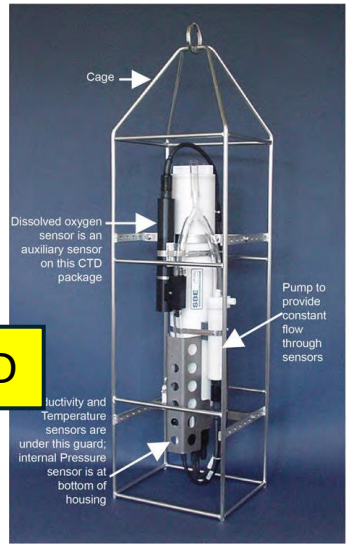
$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial v}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}$$

Estimate

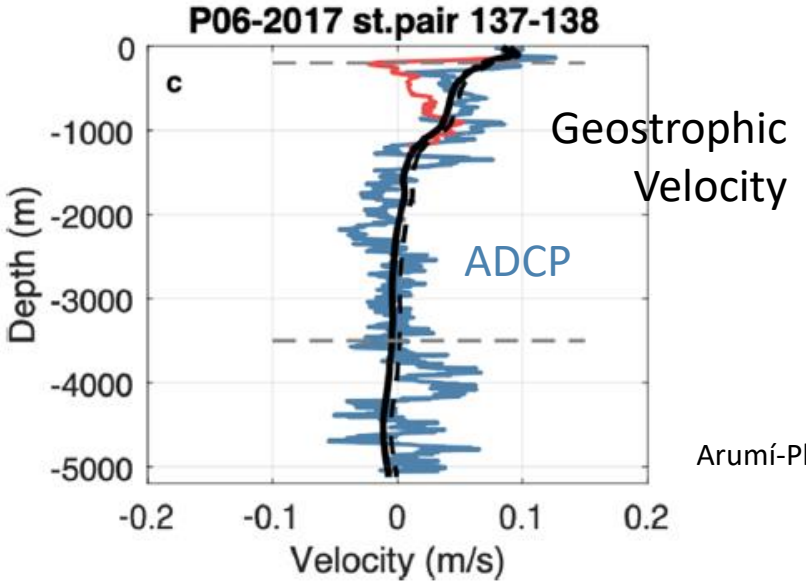
Observation

Observations (T, S, P)



CTD

Nansen water bottles before, during, and after reversing.



Arumí-Planas et al. (2022)

Data Analysis Using Models (2/2)

"State Estimation"

Wunsch, C., & Heimbach, P. (2007). Practical global oceanic state estimation. Physica D-Nonlinear Phenomena, 230(1–2), 197–208. https://doi.org/10.1016/j.physd.2006.09.040

Primitive Equations

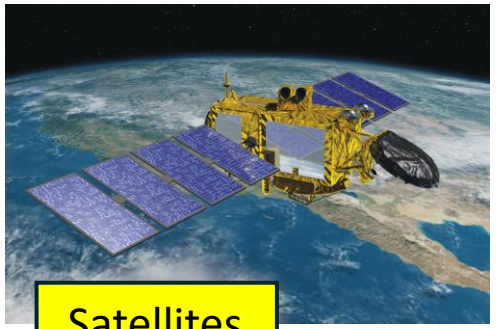
Observations/Estimates

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\nabla\cdot A\nabla u+\tau_x$$

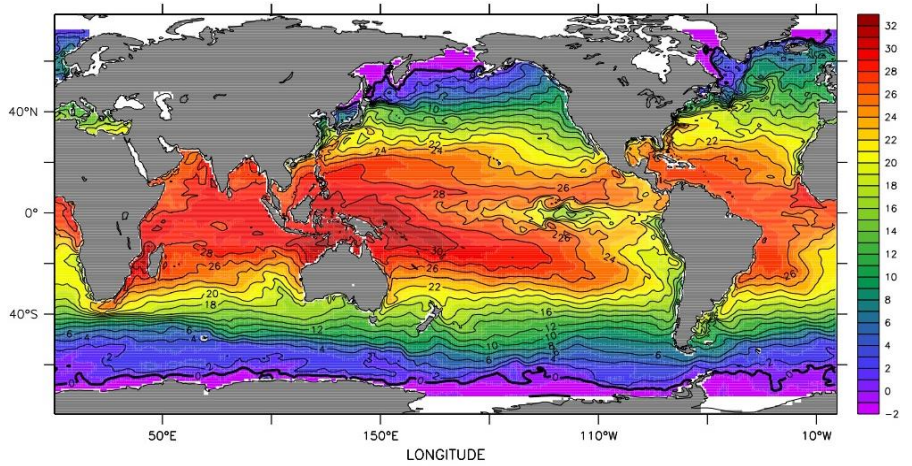
$$\vdots$$

Observations
(T, S, U, V, P)

Floats



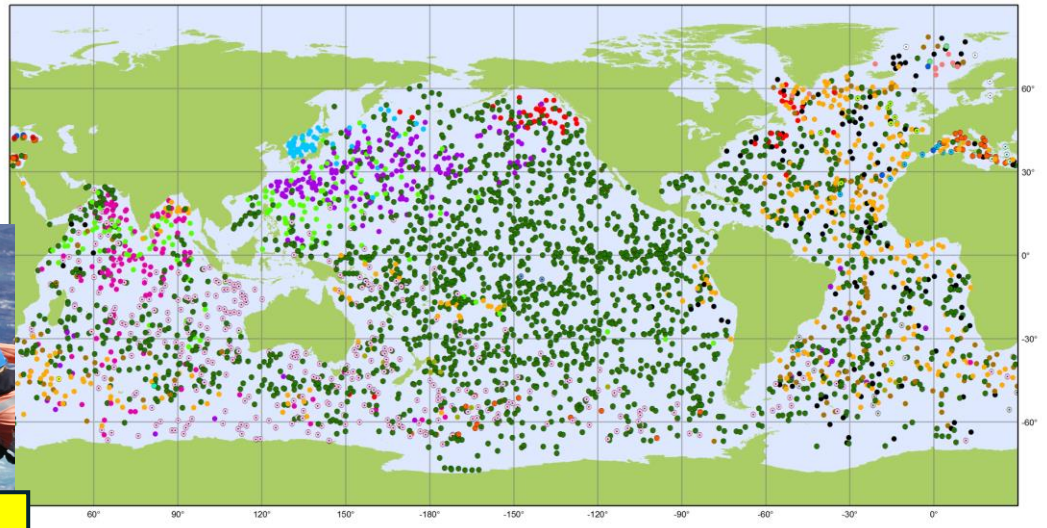
Satellites



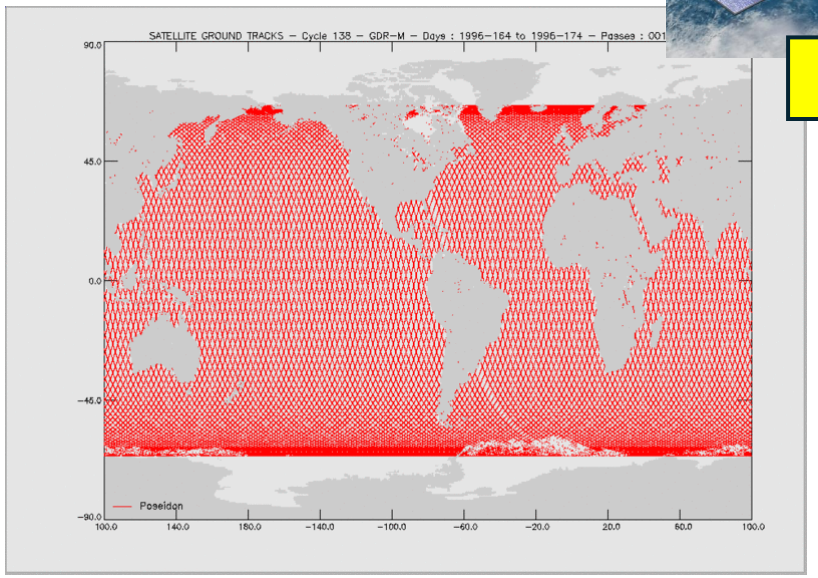
Motivator for State Estimation (1/3)

Observations are sparse, intermittent,
and are limited to a few variables.

Floats



Satellites

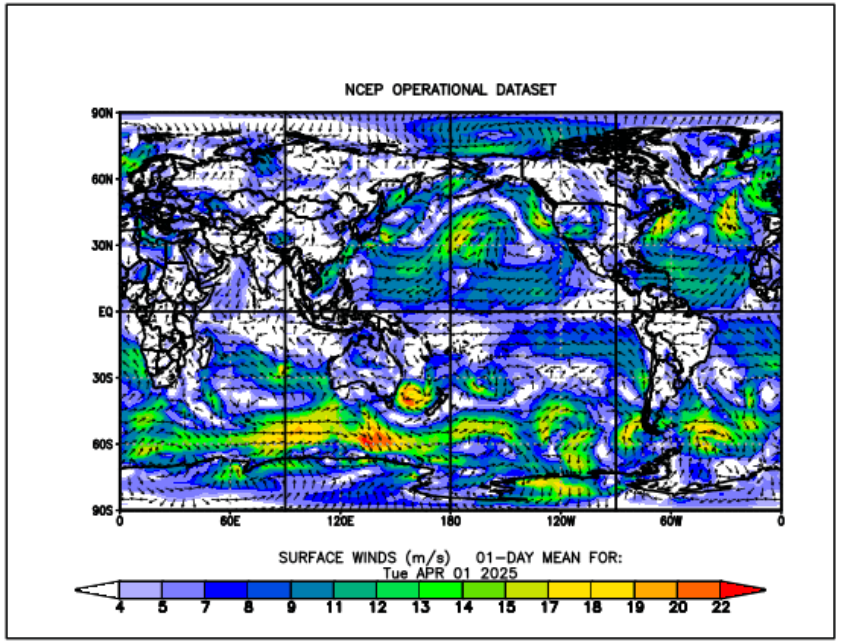


Use general circulation models to interpolate/extrapolate
the observations into complete descriptions of the ocean.

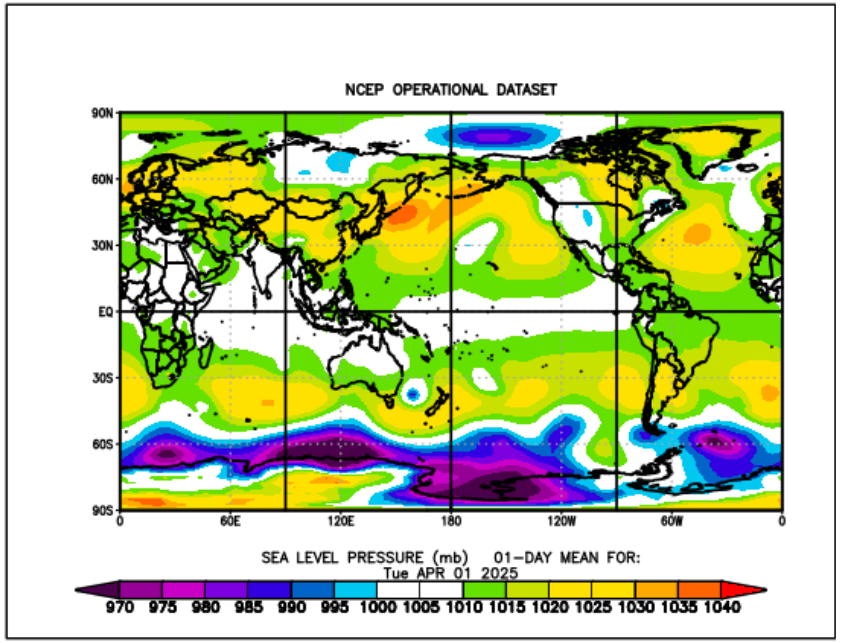
Motivator for State Estimation (2/3)

Weather Analyses (NCEP)

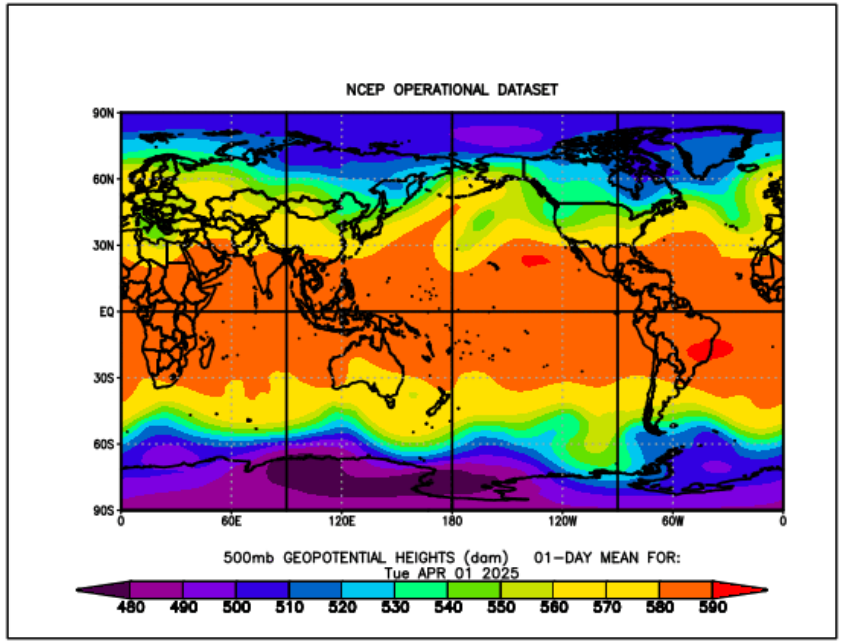
Surface Winds



Surface Pressure



500 hPa height



https://psl.noaa.gov/data/histdata/

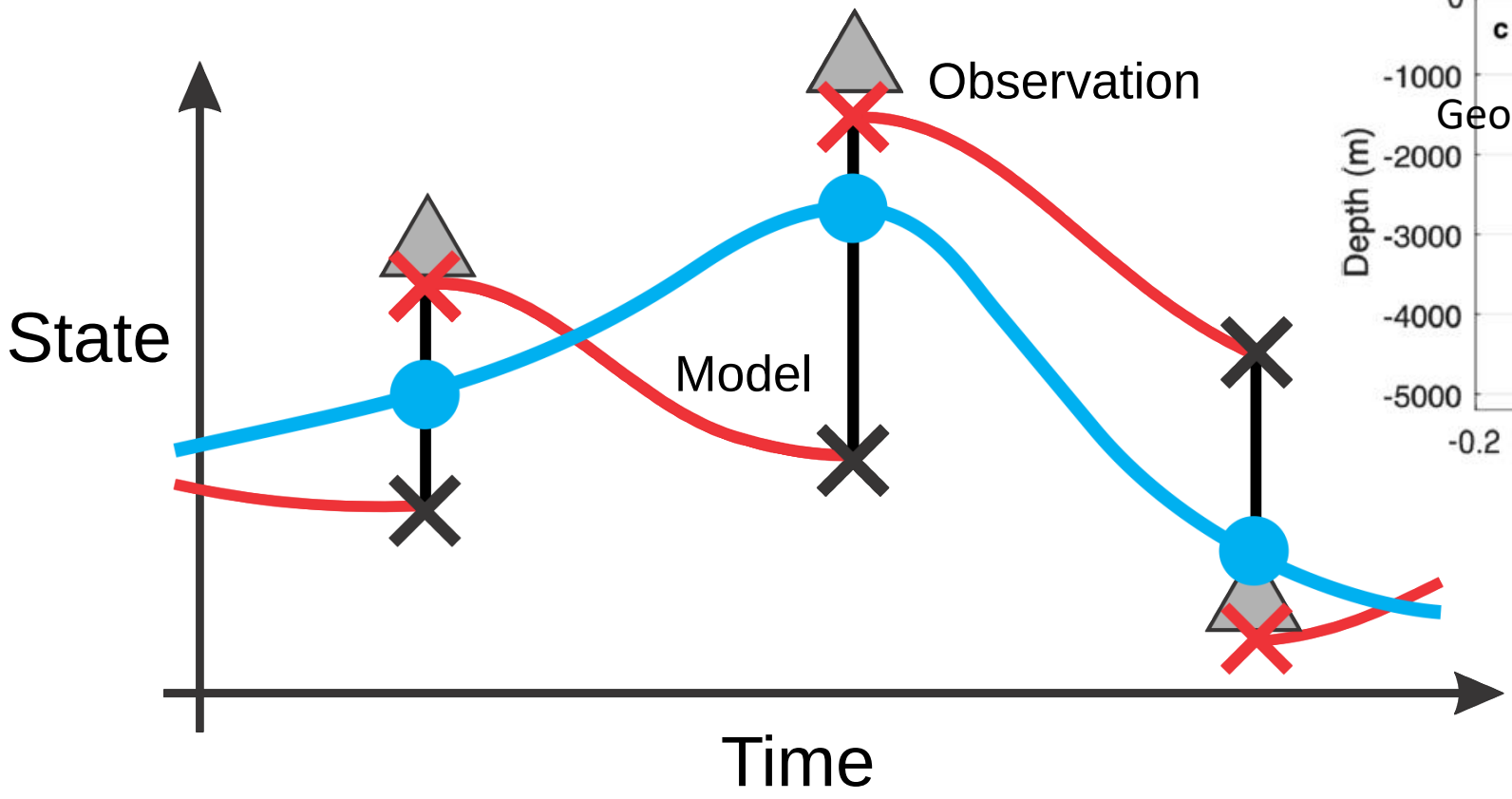
Reanalyses

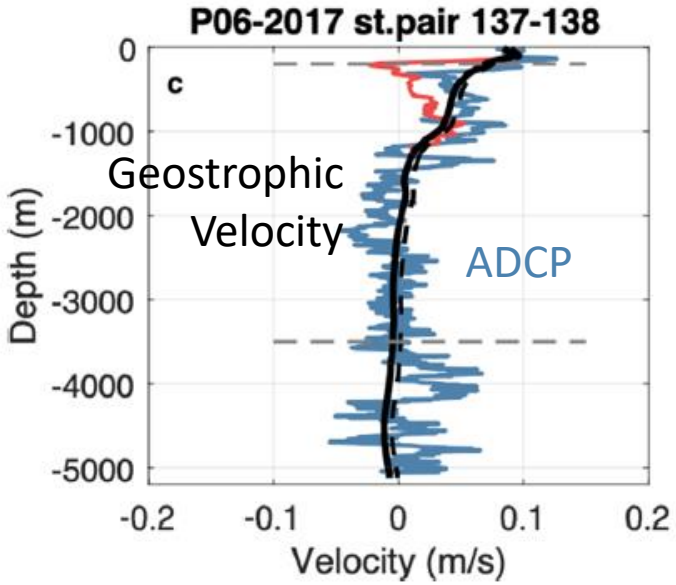
Kalnay, E., and Coauthors, 1996: The NCEP/NCAR 40-Year Reanalysis Project. Bull. Amer. Meteor. Soc., 77, 437–472, https://doi.org/10.1175/1520-0477(1996)077<0437:TNYRP>2.0.CO;2.

Motivator for State Estimation (3/3)

Data Assimilation vs State Estimation

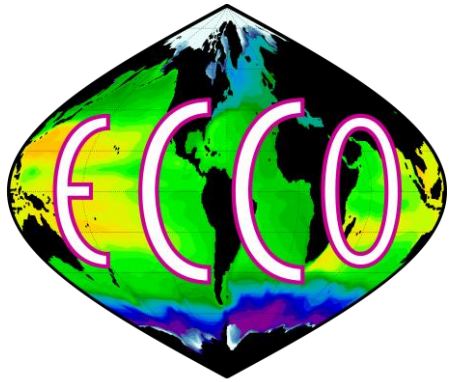
$$\frac{\partial v}{\partial z} = -\frac{g}{f\rho_0}\frac{\partial \rho}{\partial x}$$





Arumí-Planas et al. (2022)

What is ECCO?



Estimating the Circulation and Climate of the Ocean

https://ecco-group.org

A consortium to estimate ocean circulation and its role in climate that are consistent with nearly all types of observations and a state-of-the-art ocean circulation model.

Jet Propulsion Laboratory, California Institute of Technology

Massachusetts Institute of Technology

TEXAS, The University of Texas at Austin

aer, Atmospheric and Environmental Research

Scripps Institution of Oceanography

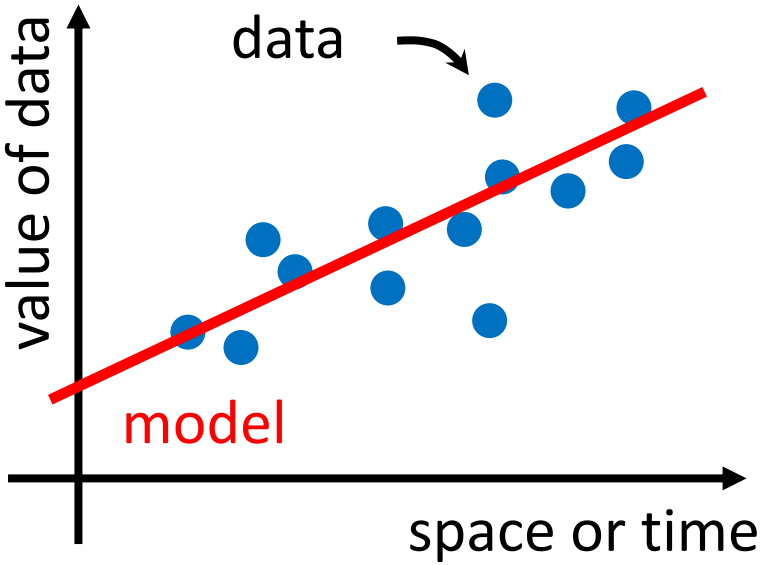
Universität Hamburg, Der Forschung | Der Lehre | Der Bildung

SJSU San José State University

AWI Alfred-Wegener-Institut, Helmholtz-Zentrum für Polar- und Meeresforschung

Woods Hole Oceanographic Institution

How State Estimation is Conducted



The problem is mathematically equivalent to curve fitting.

- Inverse Theory
- Estimation Theory
- Control Theory

Adjoint Method (aka 4D-Var)

Minimize model-data differences by adjusting the model's controls using gradients computed by the model's adjoint.

e.g., forcing, initial condition, mixing parameters

What is an Adjoint Model? (1/2)

Adjoint is a transformation used in studying mathematical relationships.

vector

scalar

Given a model $\boldsymbol{a} = \mathcal{F}(\boldsymbol{x})$ and "cost" $J \equiv J(\boldsymbol{a})$, what is $\dfrac{\partial J}{\partial \boldsymbol{x}}$?

Given the adjoint $\hat{\boldsymbol{x}} = \hat{\mathcal{F}}(\hat{\boldsymbol{a}})$ and $\hat{\boldsymbol{a}} \equiv \dfrac{\partial J}{\partial \boldsymbol{a}}$, then $\hat{\boldsymbol{x}} = \dfrac{\partial J}{\partial \boldsymbol{x}}$.

Adjoint Method (aka 4D-Var)

Minimize model-data differences J by adjusting the model's controls $\boldsymbol{x}$ using gradients $\dfrac{\partial J}{\partial \boldsymbol{x}}$ computed by the model's adjoint.

What is an Adjoint Model? (2/2)

Algorithmic Differentiation (AD)

$$\begin{pmatrix} gt \\ gtnm1 \end{pmatrix} = \begin{pmatrix} 3/2 & -1/2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} gt \\ gtnm1 \end{pmatrix} \qquad \begin{pmatrix} \hat{gt} \\ \widehat{gtnm1} \end{pmatrix} = \begin{pmatrix} 3/2 & 1 \\ -1/2 & 0 \end{pmatrix} \begin{pmatrix} \hat{gt} \\ \widehat{gtnm1} \end{pmatrix}$$

Forward Code

```
ab = 0.5*(gt – gtnm1)
gtnm1 = gt
gt = gt + ab
```

Adjoint Code

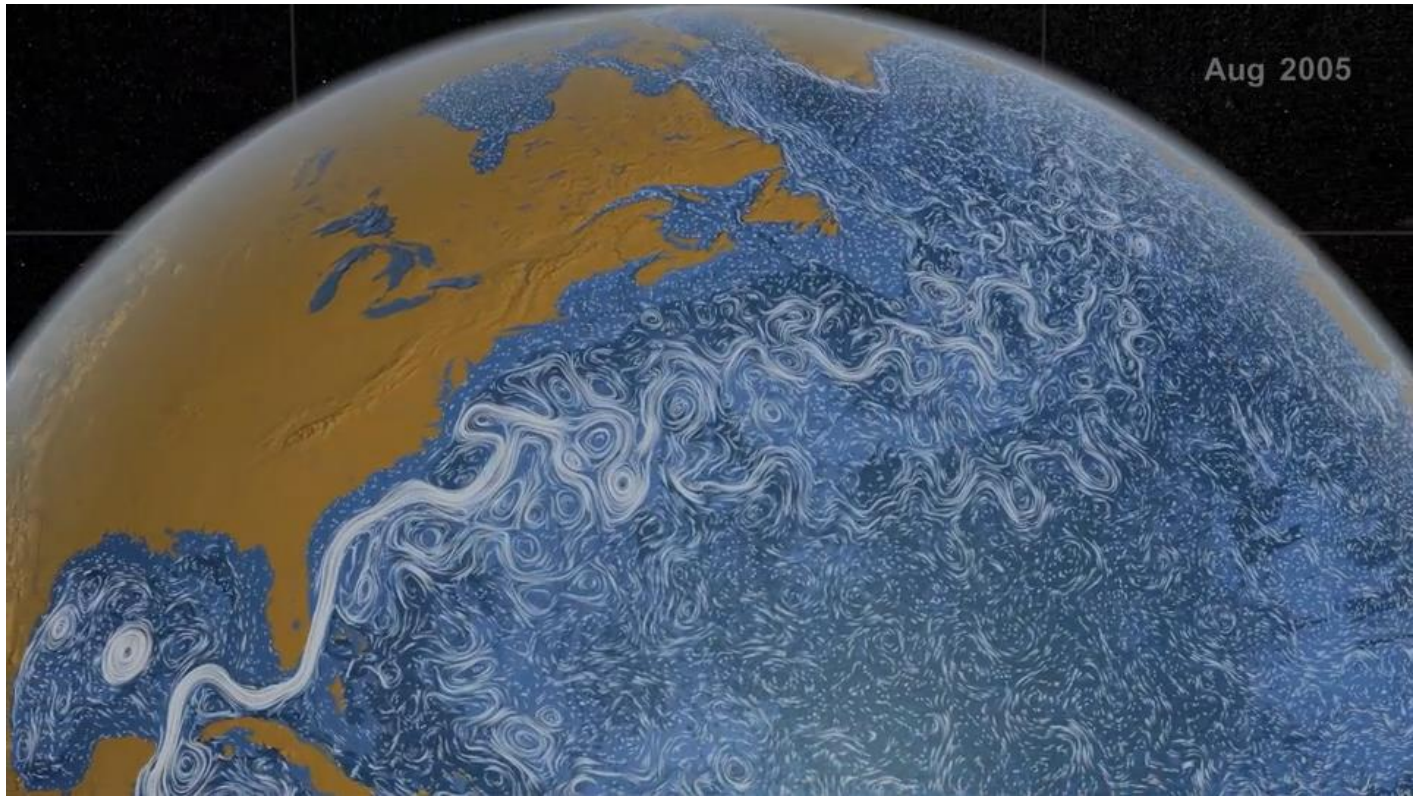
```
ab_ad = ab_ad + gt _ad
gt _ad = gt _ad + gtnm1_ad
gtnm1_ad = 0.
gt _ad = gt _ad + ab_ad *0.5
gtnm1_ad = gtnm1_ad -  ab_ad *0.5
ab_ad = 0.
```

AD Tools can automatically translate code source-to-source; e.g., TAF, Tapenade

Model used by ECCO: MITgcm

Massachusetts Institute of Technology General Circulation Model

https://mitgcm.org/

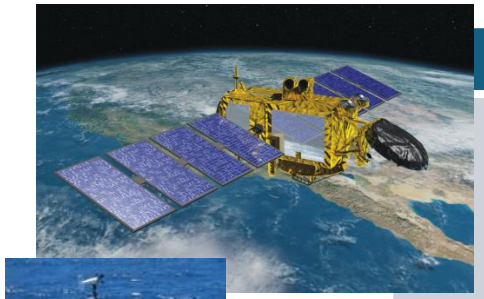


"Perpetual Ocean" (ECCO2)

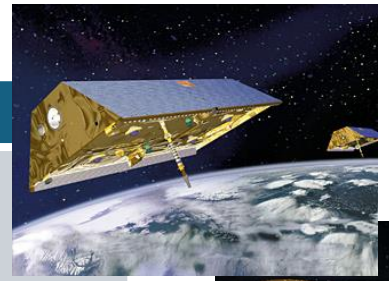
- A state-of-the-art GCM for atmosphere, ocean, and climate,
- Flexible non-hydrostatic formulation for broad applications,
- Adjoint capabilities for sensitivity studies including state estimation.

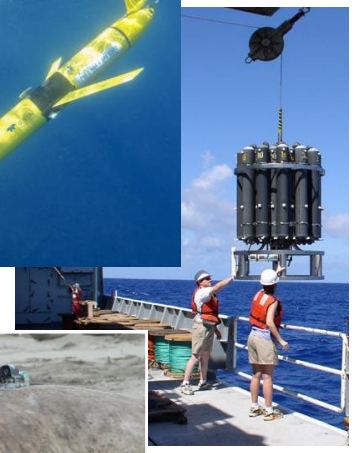
Observations Used in ECCO

ECCO Version 4 Release 4



variable	observations
Sea Level	TOPEX/Poseidon (1993-2005), Jason-1 (2002-2008), Jason-2 (2008-2017), Jason-3 (2016-2017), Geosat-Follow-On (2001-2007), CryoSat-2 (2011-2017), ERS-1/2 (1992-2001), ENVISAT (2002-2012), SARAL/AltiKa (2013-2017)
Temperature	Argo floats (1995-2017), XBTs (1992-2017), CTDs (1992-2017), marine mammals (APB 2004-2017), gliders (2003-2017), Ice-Tethered Profilers (ITP, 2004-2017), moorings (1992-2017)
Salinity	Argo floats (1997-2017), CTDs (1992-2017), APBs (2004-2017), gliders (2003-2017), ITPs (2004-2017), moorings (1992-2017)
SST	AVHRR (1992-2017)
Sea surface salinity	Aquarius (2011-2015)
Sea-Ice Concentration	SSM/I (1992-2009), SSMIS (2006-2017)
Ocean bottom pressure	GRACE (2002-2016)
TS Climatology	World Ocean Atlas 2009
Mean Dynamic Topography	DTU17 (1993-2015)





ECCO State Estimates

https://ecco-group.org/products.htm

ECCO Central Estimate

ECCO's flagship multi-decadal global ocean circulation estimate

ECCO-Darwin

Global eddy-permitting coupled biogeochemical-ocean estimate

ASTE

Eddy-permitting Arctic-North Atlantic estimate

B-SOSE

Eddy-permitting coupled biogeochemical-ocean estimate of the Southern Ocean

CASE, TPOSE, GoM, NWPAC

Eddy-resolving regional estimates

What Good is ECCO's State Estimate? (1/4)

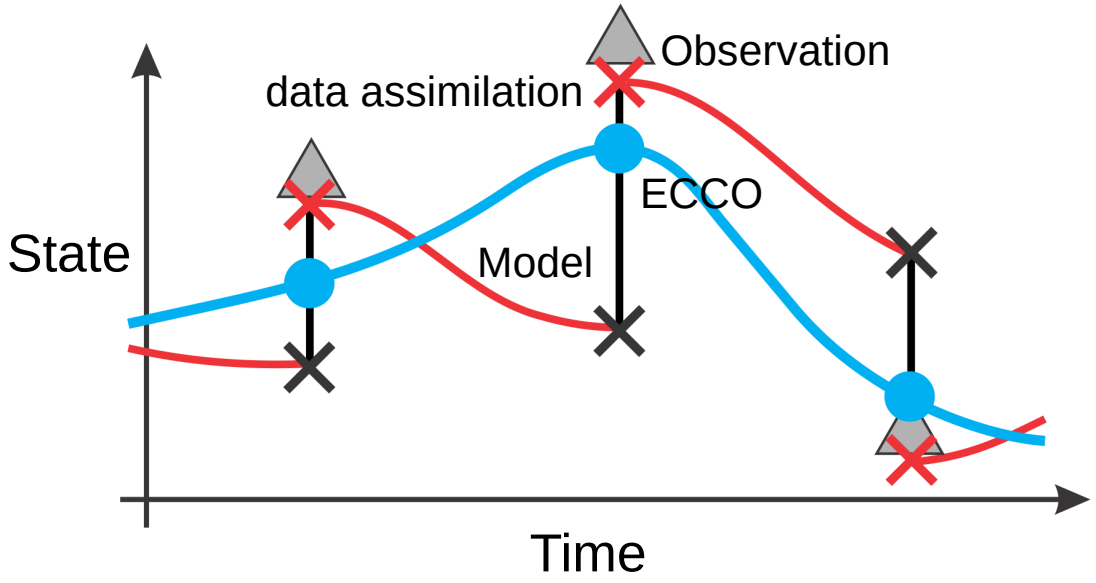
ECCO provides complete descriptions of the ocean.

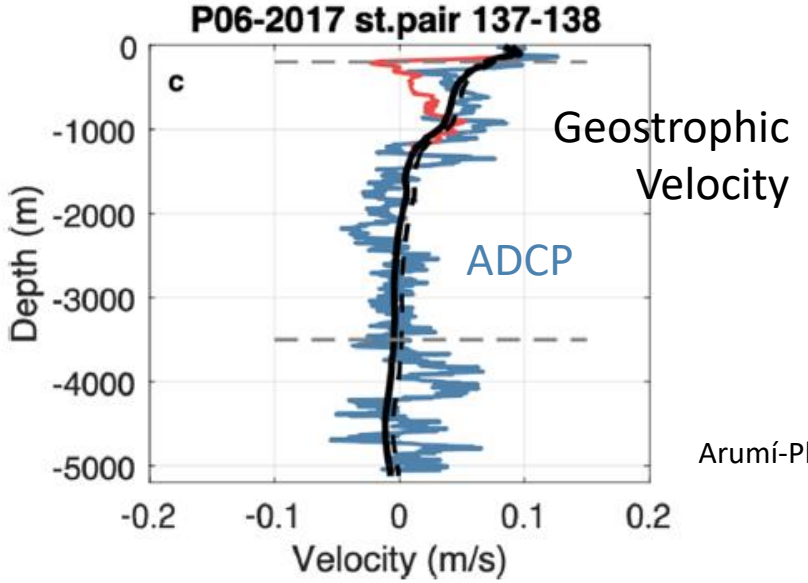
"Standard Output"	• Monthly mean UVTS, ssh, OBP • Daily mean ssh, OBP
"Extended Output"	• Control estimates (e.g., forcing) • Fluxes for budget analysis • Re-gridded latitude-longitude fields • Daily mean UVTS

What Good is ECCO's State Estimate? (2/4)

Estimates are largely consistent with observations.

Estimates do not necessarily match observations because of uncertainties, especially representation error.



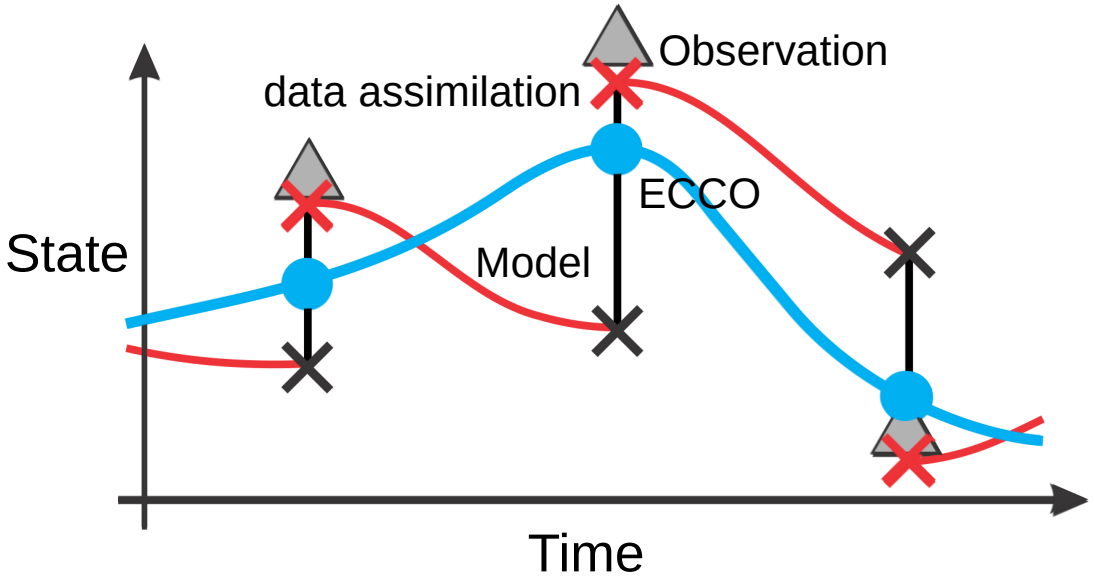


Arumí-Planas et al. (2022)

Examples: missing model physics (ageostrophic motion, subgrid-scale variations), errors caused by unadjusted controls.

What Good is ECCO's State Estimate? (3/4)

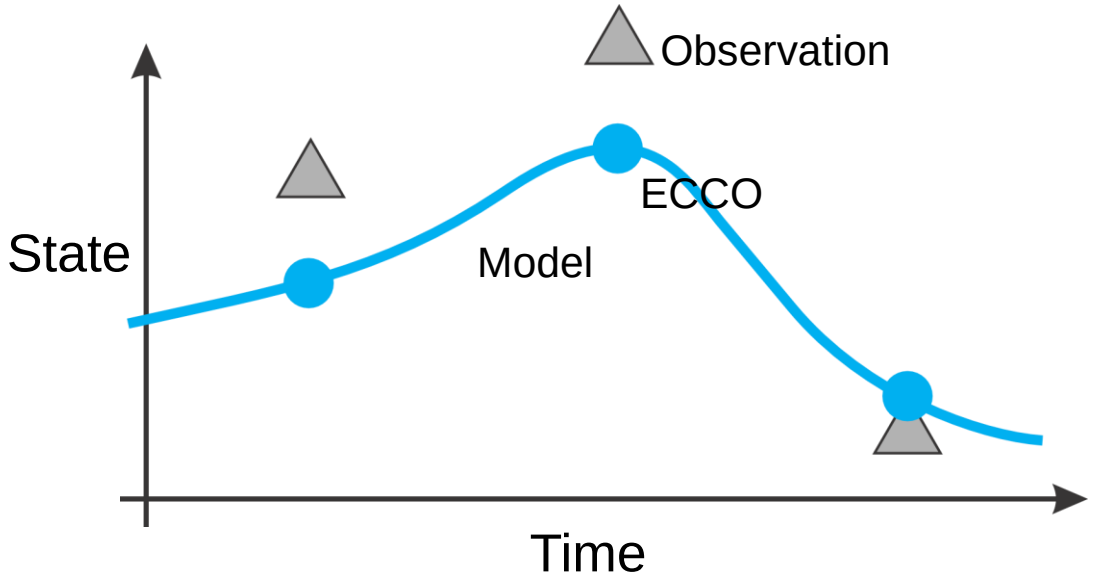
Estimates are physically consistent.



- Estimated state is a solution of the model (e.g., non-divergent circulation),
- Property budgets can be closed,
- The physics controlling the state is described by the model.

What Good is ECCO's State Estimate? (4/4)

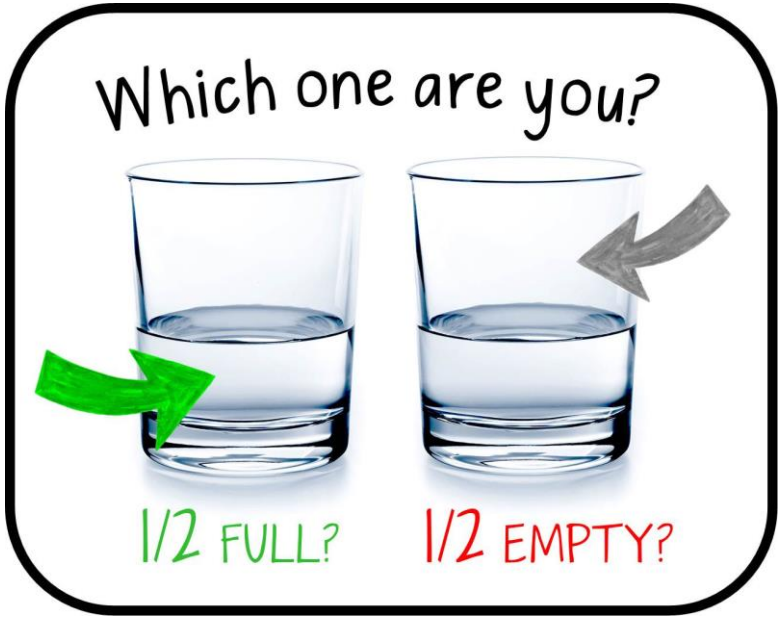
Why should you trust the solution?



- Estimates are based on first principles under clearly stated assumptions,
- Observations limit undue corrections to model controls,
- The solutions being consistent with nearly all extant observations gives confidence in the estimates' fidelity,
- The scientific method dictates that an estimate that is consistent with both observations and theory cannot be dismissed.

Where do we go from here?

- Analyze state output
- Investigate budgets
- Interrogate model
- Exploit adjoint



- Improve model
- Add more observations
- Extend analysis period
- Refine optimization

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