Bottom Topography as a Control Parameter in an Ocean Circulation Model

Martin Losch and Carl Wunsch

Department of Earth, Atmospheric, and Planetary Sciences, MIT, Cambridge

(email: mlosch@mit.edu)

1 Overview

Bottom topography, $h(x,y)$, is a major factor in determining the general circulation of the ocean. It is, however, inaccurately known in many regions, and even where accurately known, the best way to represent (parameterize) it in models is obscure. To begin to understand the influence of errors in h and of misrepresentations of both resolved and sub-grid scale structures, a linear barotropic shallow water model and its adjoint are developed in which depth is used as a control variable. Simple basin geometries are employed to explore the extent to which topographic structure determines the sea-surface elevation in a steady flow and, more directly, the information content about the bottom contained in elevation measurements.

Experiments show that even perfect measurements of sea-surface elevation in a steady state cannot, by themselves, uniquely determine the full structure of h. (There is a null space.) As in most control problems, a priori knowledge of its structure is useful in the best topographic determination. Resolution of h as a function of position is greatest where the flow velocities are greatest. Spatial correlation between the resolution of h and the flow field is weaker (as expected) when noise with realistically large variance is introduced into the data.

Ultimately, bottom topography will likely be included generally as a control variable in GCMs of arbitrary complexity along with other controls such as friction and lateral boundary conditions.

2 Shallow Water Model and Adjoint Method

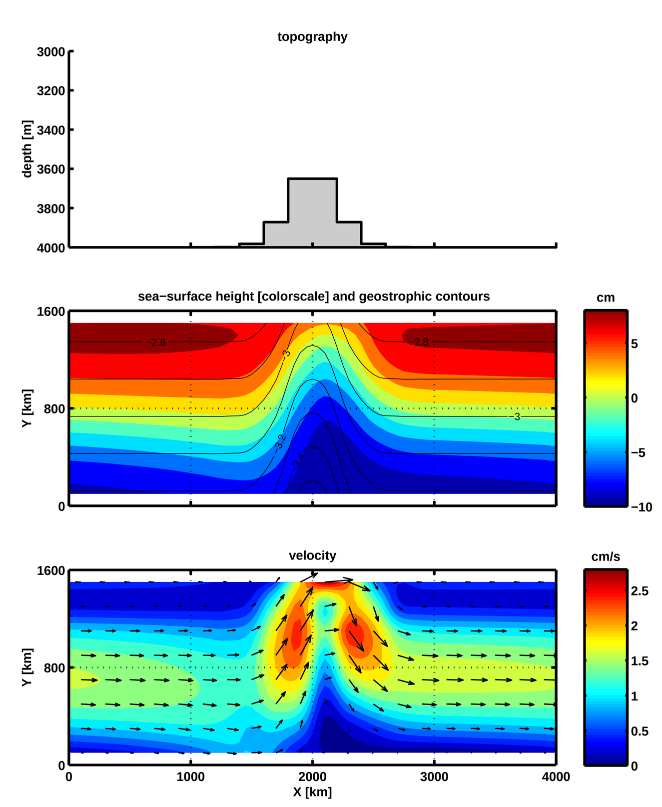

Figure 1: Steady-state flow field of linear shallow water equations on β-plane with linear parameterization of bottom stress and zonal wind stress $\tau^{(y)} = \sin \pi y/Y$ in a zonal channel on the southern hemisphere over idealized topography (Gaussian sill). **Top:** zonal section along channel with topography, **middle:** sea-surface height in cm, **bottom:** velocity in cm/s.

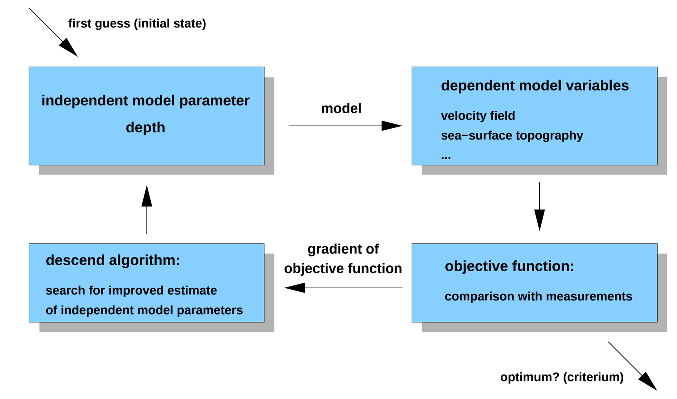

Figure 2: Schematic of the iterative minimization algorithm. The adjoint model is generated by an automatic differentiation tool (TAF, Giering and Kaminski, 1998).

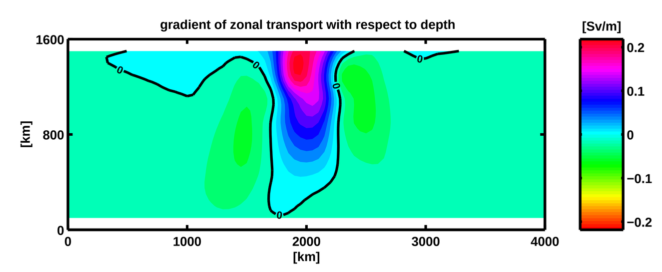

Figure 3: First application of the adjoint model: gradient of the total volume transport through the channel with respect to depth, h. The sensitivity of the flow is largest where the current speed is high (compare with Fig. 1)

3 Perfect Data and Prior Error Estimates

A least-squares fit of the model to perfect data with the objective function

$$\mathcal{J}_1 = \frac{1}{2} \sum_{\text{all data}} (\eta - \eta_d)^2 / \sigma_\eta^2,$$

shows that all but $2\Delta x$-grid scale components of the topography are formally observable. These nullspace components are due to the numerical scheme of the model.

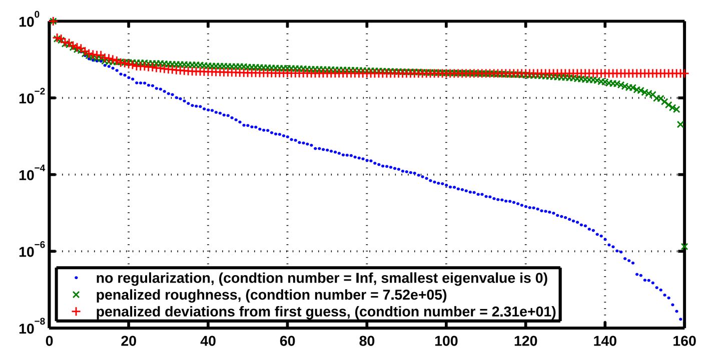

Figure 4: Eigenvalues of Hessian matrix of different objective functions. Penalty terms ensure convergence of the minimization algorithm and a Hessian matrix with finite condition number. Without these terms, the spectrum has at least one eigenvalue that is numerically zero (corresponding to $2\Delta x$-waves). Penalizing deviations from the first guess topography leads to a smaller condition number of the Hessian matrix (faster convergence) than penalizing roughness of the solution.

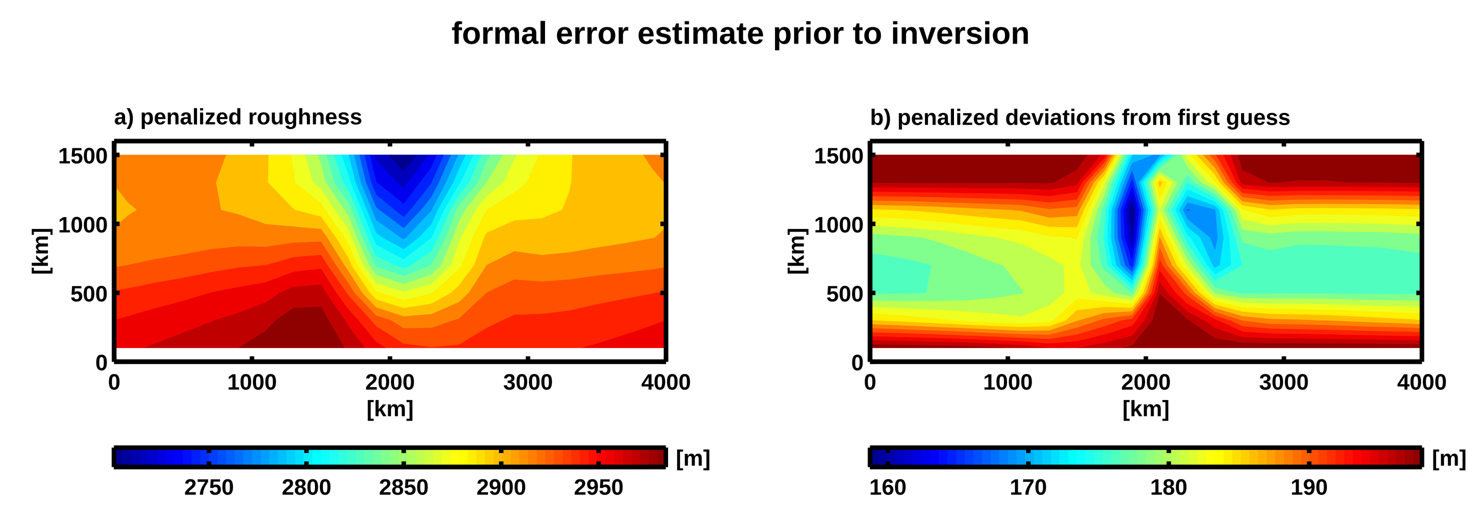

Figure 5: With objective function

$$\mathcal{J}_2 = \mathcal{J}_1 + \frac{1}{2}(\mathbf{h} - \mathbf{h}_0)^T \mathbf{W} (\mathbf{h} - \mathbf{h}_0)$$

we introduce prior knowledge of the topography's structure. At the same time the model-data misfit is no longer perfect. $\mathbf{W}$ is the inverse of a prior covariance matrix, $\mathbf{h}_0$ a prior estimate of the topography. The prior sea-surface height error estimate σ_η is constant in space and time. Its value is $\sigma_\eta = 10$ cm according to the average combined error of satellite altimetry and an underlying geoid model (Wunsch and Stammer, 1998). **a)** By choosing $\mathbf{h}_0 = 0$ and off-diagonal terms for $\mathbf{W}$, we seek smooth solutions. Small eigenvalues (compare Fig. 4) dominate the formal posterior error estimate for depth (square-root of the diagonal of the inverse Hessian matrix). **b)** The choice of $\mathbf{W} = \sigma_h^{-2}\mathbf{I}$ and a high prior error of depth of $\sigma_h = 200$ m weakly penalizes deviations from the initial guess $\mathbf{h}_0$. In this case, the formal posterior error is smallest where current speeds are high.

4 Recovering Depth from Sea-Surface Height Data and a (False) Initial Guess

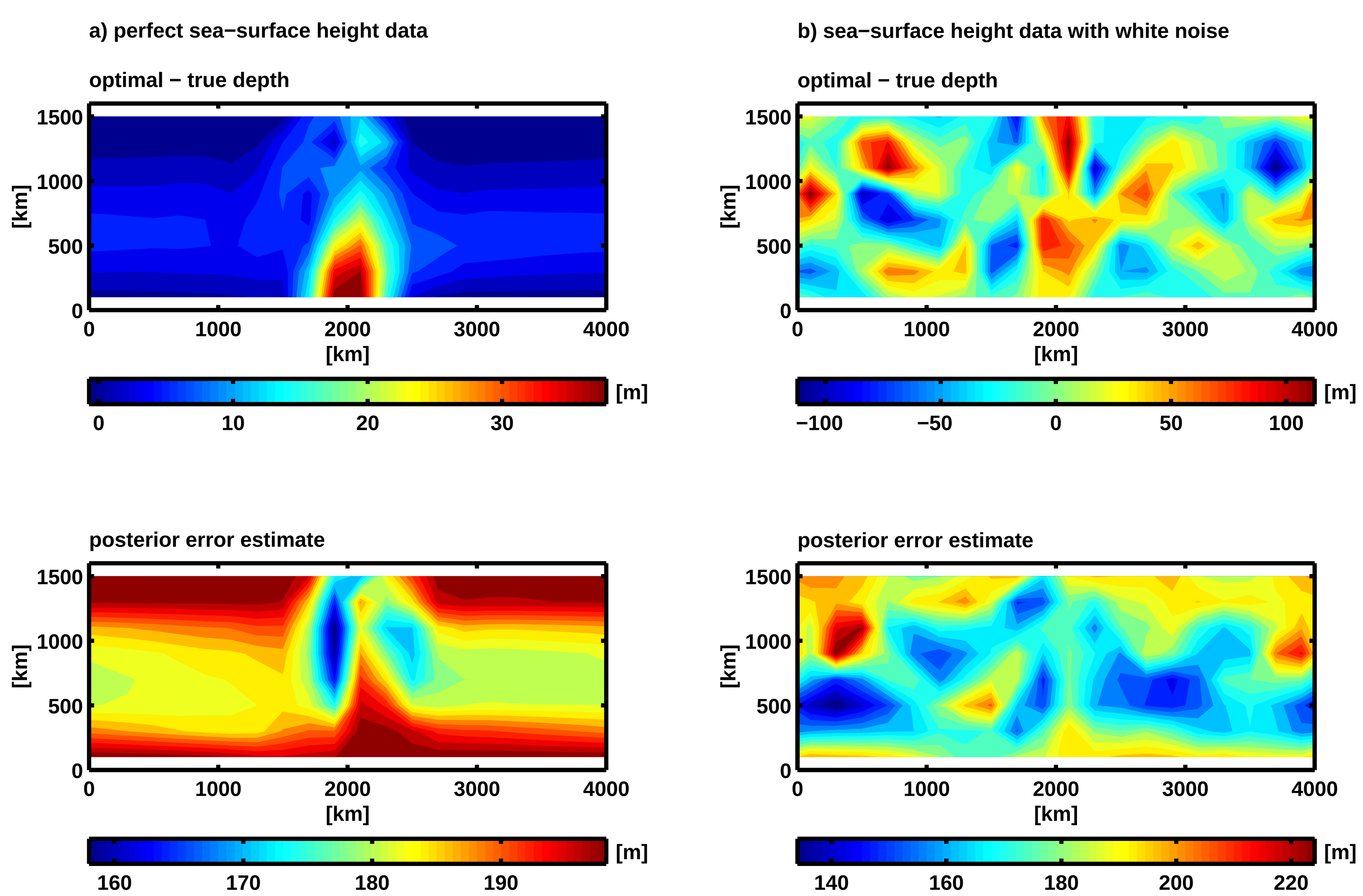

Figure 6: Solution of optimizing an objective function that penalized deviations from the initial depth estimate. Top row shows the difference between optimal depth estimate and true depth, bottom row is the posterior error estimate of the optimal depth. **a)** Perfect sea-surface height data, however with a finite weight of $1/\sigma_\eta^2 = 1/(10\text{ cm})^2$, rms(optimal estimate – true depth) = 9 m, **b)** sea-surface height data plus white noise with a standard deviation of 10 cm, rms(optimal estimate – true depth) = 45 m.

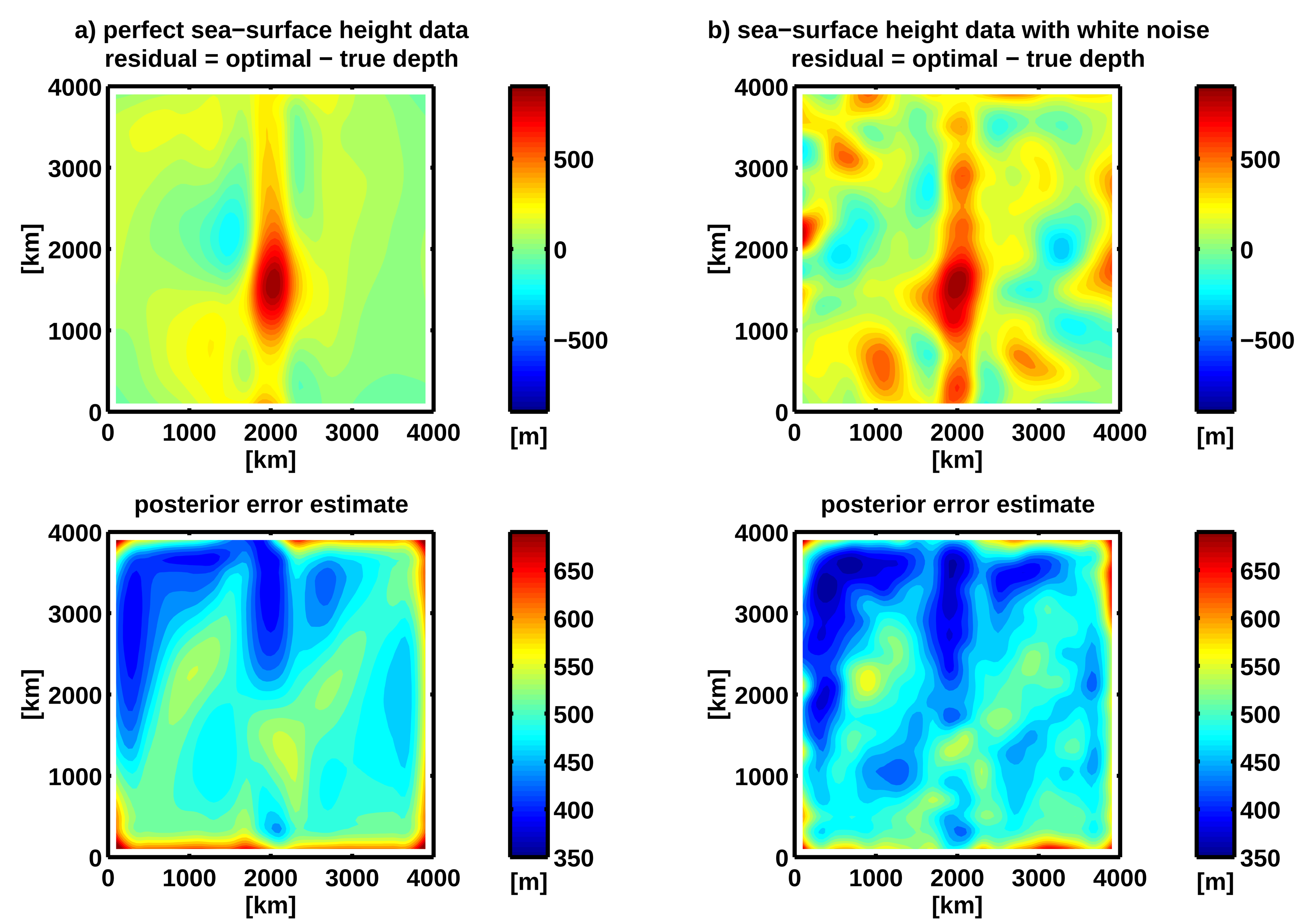

Figure 7: Repeating the exeriments with a subtropical gyre setup with a "mid-Atlantic ridge" of 2000 m height shows that it is more difficult to obtain an accurate estimate of the true depth in this configuration. The flow field does not depend as much on the topography as in the channel experiments.

5 Discussion and Outlook

- Sensitivity of the flow to topography is largest where current speeds are high.
- Prior knowledge about topography is necessary to overcome a numerical nullspace; a bias towards the (false) initial guess of topography leads to better convergence and to a more accurate depth estimate than penalizing roughness.
- The solution is sensitive to noise in the sea-surface height data with realistic amplitude.
- The flow field in the zonal channel is more sensitive to changes in topography than in a subtropical gyre flow.

We present results from on-going work. Future delevopments will include:

- using different domains;
- inclusion of time dependence, baroclinic effects, more realistic topography;
- use of different types of data;
- optimization of depth in a full ocean general circulation model.

References

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